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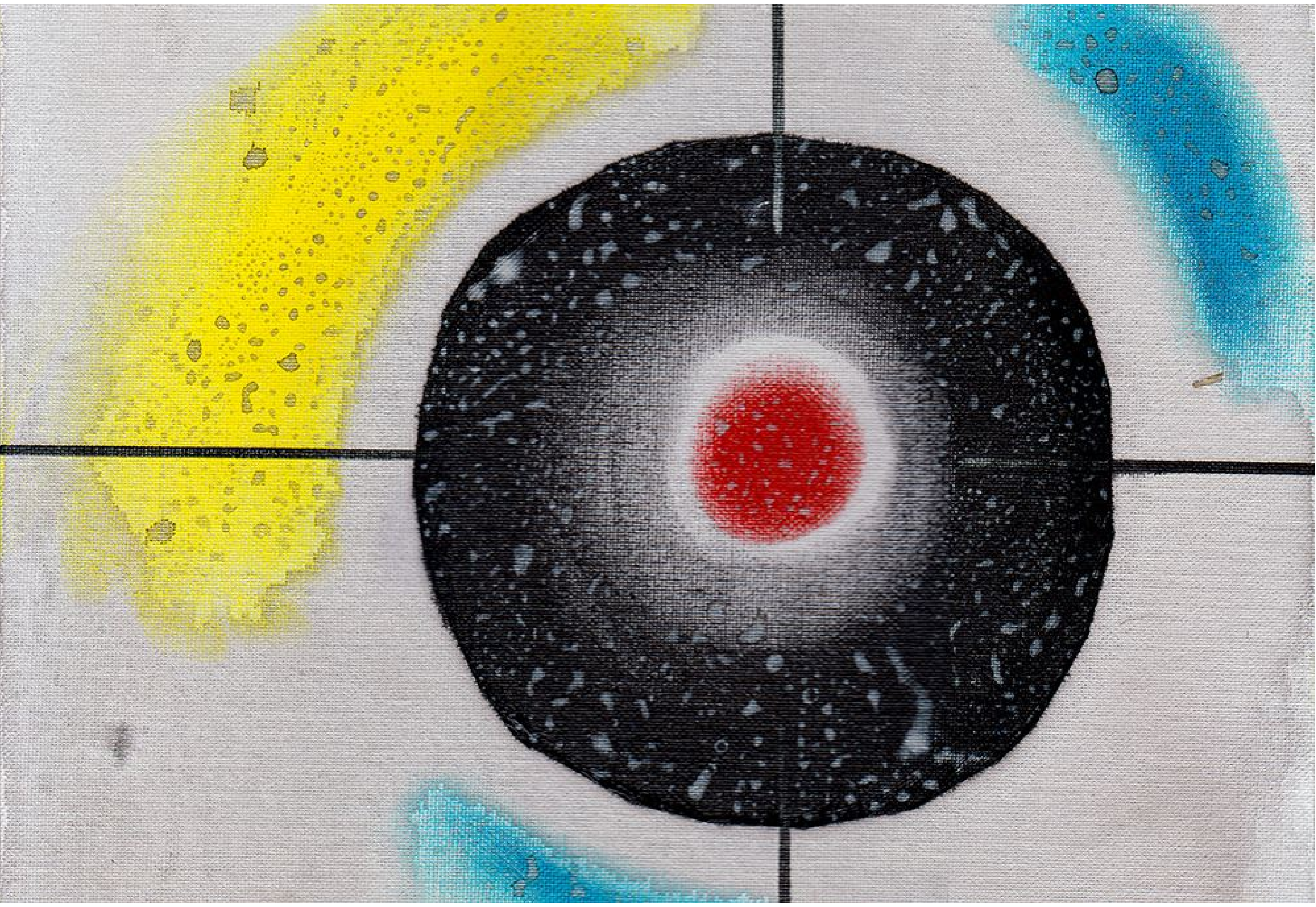
WEALTH DISTRIBUTION AND MONETARY POLICY



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CONTENTS

ABBREVIATIONS

AR – autoregressive process
BGP – balanced growth path
bps – basis points
BVAR – Bayesian vector autoregression
CESEE – Central, Eastern and Southeastern Europe
DNK – dynamic New Keynesian model
ESCB – European System of Central Banks
EU – European Union
Eurostat – statistical office of the European Union
EU-SILC – European Survey of Income and Living Conditions
HFCS – Household Finance and Consumption Survey
HICP – Harmonised Index of Consumer Prices
IES – intertemporal elasticity of substitution
IMF – International Monetary Fund
IRF – impulse response function
IS – investment-savings model
MPC – marginal propensity of consumption
OAD – old-age dependency
OLG – overlapping generation
UK – United Kingdom
US – United States of America

SUMMARY

We observe differences in the net wealth distribution by age among European countries. The net wealth distribution in Western EU countries is consistent with the life cycle hypothesis. However, in Eastern EU countries, the wealth distribution is skewed towards younger ages. The aim of the paper is twofold: first, we study the characteristics of economies leading to differences in the net wealth distribution by age; second, we evaluate the impact of these differences on the transmission of monetary policy. To do so, we develop a modified New Keynesian model where the demand side is represented by a multi-period overlapping generation setup, and the supply side of the economy follows the New Keynesian framework. The model is used to analyse the interaction between monetary policy and wealth accumulation originated by demographics and the productivity gap among generations in a coherent general equilibrium model. The HFCS database is used to calibrate the model for two groups of European countries. We find that the shape of net wealth distribution by age has an important bearing on the effectiveness and hence conduct of monetary policy.

Keywords: overlapping generations model, New Keynesian model, wealth distribution, monetary policy

JEL codes: E32, E52, J11

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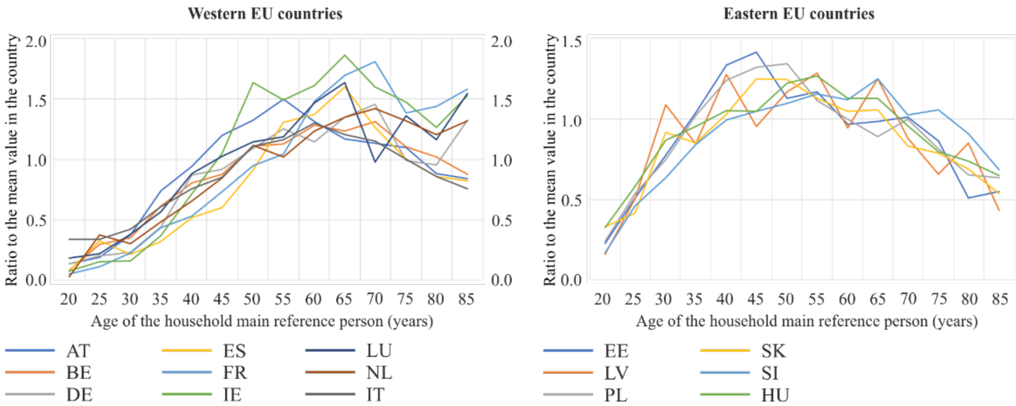
1. INTRODUCTION

The net wealth distribution by age differs for some European countries. In Austria, Belgium, Cyprus, Germany, Spain, Finland, France, Greece, Ireland, Italy, Luxembourg, Malta, the Netherlands and Portugal (hereafter, Western EU countries), the majority of net wealth is held by the agents who are at their retirement age which is consistent with the life-cycle theory of Modigliani and Brumberg (1954). On the other hand, in Hungary, Latvia, Estonia, Poland, Slovenia and Slovakia (hereafter, Eastern EU countries), net wealth is distributed by age and is skewed towards younger ages. This is clearly illustrated in Figure 1, which displays the ratio of net wealth by age to the mean value of total net wealth in a country.

Figure 1
Ratio of net wealth by age to the country



Source: Authors' estimations using HFCS wave 2 data.

In the present paper, we seek to investigate whether differences in the shape of the net wealth distribution by age affect the effectiveness of monetary policy. In this context, it is crucial to identify the source(s) of differences in the wealth distribution by age. Accordingly, the aim of this paper is twofold: first, we explore the possible explanations and reasons for differences in the net wealth distribution by age in these countries. Next, we develop a coherent theoretical model, which incorporates these features, to analyse the impact of monetary policy shocks.

Cross-country differences in net wealth distribution by age could be explained by several factors, such as demographic profiles, social, cultural and historical backgrounds of countries, households' preferences and institutional differences. Among these factors, we observe significant differences in the historical backgrounds and age structure of these two groups of countries. Until the end of the 20th century, Eastern EU countries were part of the centrally planned economic system. Due to the collapse of this system, we observe discrepancies in the productivity level between cohorts.[1] Some implicit barriers hinder old workers from starting making use of the novel resources bred by the new system, which contributes to their productivity. Therefore, younger individuals earn higher labour income compared to their older colleagues. We call this kind of heterogeneity in productivity levels among cohorts – *generation heterogeneity*. A higher wage income at the early stages of lifetime of the young agents implies faster accumulation of wealth. In Western EU countries, we do

[1] Lovász and Rigó (2013) identify the existence of the productivity gap among generations in Hungary by using an employer-employee data set covering the years 1986–2008.

not observe such significant productivity differences between generations, thus the accumulation of net wealth reaches the maximum level at the retirement age. In the following sections of the paper, we provide an evidence supporting our assumption of *generation heterogeneity* at productivity levels.

Another factor, which could lead to differences in the net wealth distribution, is the age structure of the country. In Western EU countries, OAD ratios, the ratio of retired agents to workers, are generally higher. In these countries, the number of old agents is high and the majority of net wealth is held by the older individuals. Due to both factors, the wealth distribution skews towards older ages. On the other hand, the OAD ratios have been relatively lower in Eastern EU countries for the last 30 years, resulting in the peak of the net wealth distribution being skewed towards younger generations. We are aware of the fact that the productivity gap between generations in Eastern EU countries will eventually disappear, and the OAD ratios will decline as in Western EU countries. Therefore, in the long term, the shape of the Eastern and Western EU countries' wealth distribution will look more alike. However, in the meantime, there are discrepancies in the wealth distribution due to the differences in demographics and productivity levels. By taking a snapshot of today, we are interested in the impact of differences in the wealth distribution on the effectiveness of monetary policy.

Since the wealth distribution in these two groups of countries is not similar, we expect different responses to the same monetary policy shock. In this context, it is crucial to dissect the transmission channels of monetary policy: *wealth*, *substitution* and *income effects* of monetary policy. If the majority of wealth is held by older age groups, monetary policy is expected to have a lower impact on the economy, and since old agents' income relies more on financial assets, an increase in the interest rate will lead to a higher increase in old agents' wealth. Meanwhile, the life expectancy of retired people is lower and hence their MPC is higher than that of younger individuals. Consequently, old agents will consume the additional interest income instead of saving it for tomorrow. Therefore, the overall response of the output and inflation will be weaker compared to the economy in which the majority of the wealth is held by younger or middle-aged agents. Accordingly, the *wealth effect* of monetary policy shock becomes stronger as the net wealth distribution moves towards older ages. The impact of monetary policy mitigates even more if OAD is high in the economy. Basically, the mechanism can be summarized as follows: young agents consume and save their labour income. On the other hand, old agents consume their savings made from interest income when they were young. Accordingly, these agents respond differently to unexpected monetary policy shocks in line with their objectives. For instance, after a tightening monetary policy shock, young agents' level of consumption decreases due to the stronger *substitution effect*, and that of older agents increases owing to the stronger *wealth effect*. As the OAD ratio increases, the dominance of the *substitution effect* weakens over the *wealth effect*. In addition to these channels, tightening monetary policy has a negative impact on the labour demand, followed by a decline in the labour income. We call this channel *income effect*.

This paper is related to several strands of the monetary policy literature. First of all, most recently, Kantur (2013), Wong (2016), Bielecki et al. (2018), Berg et al. (2019) and Leahy and Thapar (2019) have traced the heterogeneous effect of monetary policy on individuals at different age levels. Wong (2016) and Berg et al. (2019) particularly concentrate on the consumption level of different age groups and show that the effectiveness of monetary policy is stronger for the young agents. There are also papers focusing on the same issue from different perspectives. For example, Leahy and Thapar (2019) and Selezneva et al. (2015) suggest that the middle-aged households benefit the most from the expansionary monetary policy because the debt burden is the highest at that age. This finding is in line with the studies by Cloyne et al. (2018) and Calza et al. (2013). Both papers show that households with mortgages are more responsive to monetary policy, especially in the case of flexible interest rates.

Furthermore, this paper contributes to the literature on the effectiveness of monetary policy in different European countries. Studies, such as Feldkircher and Huber (2016), Fadejeva et al. (2017), Burriel and Galesi (2018) and Hajek and Horvath (2018), show that responses to the changes in the policy rate in Central and Eastern EU countries are larger than in Western EU countries. This paper provides an explanation for the discrepancies between responses in these countries, using the shape of the net wealth distribution. We show that if the net wealth is skewed towards young ages due to the productivity gap among cohorts, it might partly explain stronger volatility of impulse responses in Eastern EU countries compared to Western EU countries.

Another strand of literature, which focuses on the impact of monetary policy on the redistribution of wealth and is closely related to our research, is the microsimulation analysis of a monetary policy shock. Ampudia et al. (2018) shows that monetary policy generates heterogeneous effects on euro area households depending on the composition of their income and on the components of their wealth. They distinguish channels of direct and indirect effects: direct effects can be heterogeneous across households depending on their initial wealth, but the indirect effects operate through responses of prices and wages, hence of labour income and employment. By using the HFCS data collected from 25 EU countries, the authors show that the indirect effect prevails over other channels by responding to the innovations in conventional and unconventional monetary policy in the euro area.

In order to explore the impact of the shape of wealth distribution on the effectiveness of monetary policy, we develop a modified New Keynesian model, which merges multiple-period OLGs and DNK frameworks with the above characteristics of Eastern and Western EU economies.[2] The demand side of the model assumes an OLG structure, which enables us to introduce the aforementioned *generation heterogeneity* and different OAD ratios into the model economy. Following the standard DNK setup of Galí (2015), the introduction of price rigidities to the model allows monetary policy to influence interest rates and the real economy. This highly stylized model enables us to study the impact and the transmission of monetary policy in economies with a different net wealth distribution. Furthermore, this framework enables us to analyse the impact of monetary policy shock on aggregate and cohort specific variables in the model economy.

The main findings of the paper can be summarized as follows: first, at aggregate level, the effectiveness of monetary policy on output and inflation weakens as the net wealth distribution moves towards older ages. Second, at individual level, monetary policy is more effective for younger agents of low OAD ratio economies with a productivity gap between generations. Third, generation heterogeneity determines the skewness of

[2] The model used in this paper is an extension of Kantur (2013). A similar setup without a productivity gap assumption is used in studies, such as Fujiwara and Teranishi (2008; 2005), Bielecki et al. (2018), Kara and von Thadden (2016).

the net wealth distribution. Moreover, the wealth distribution has an impact on the responses at both individual and aggregate levels. Adding to the findings of Kantur (2013), Carvalho et al. (2016), Kara and von Thadden (2016), Wong (2016) and Bielecki et al. (2018), we show that the natural rate of interest decreases monotonically not only as the OAD ratio increases, but also as the productivity gap among generations disappears and net wealth is skewed towards older generations. Finally, findings of the paper suggest that the stronger reaction of Eastern EU countries to a monetary policy shock as compared to Western EU countries can be partly attributed to the differences in the net wealth distribution.

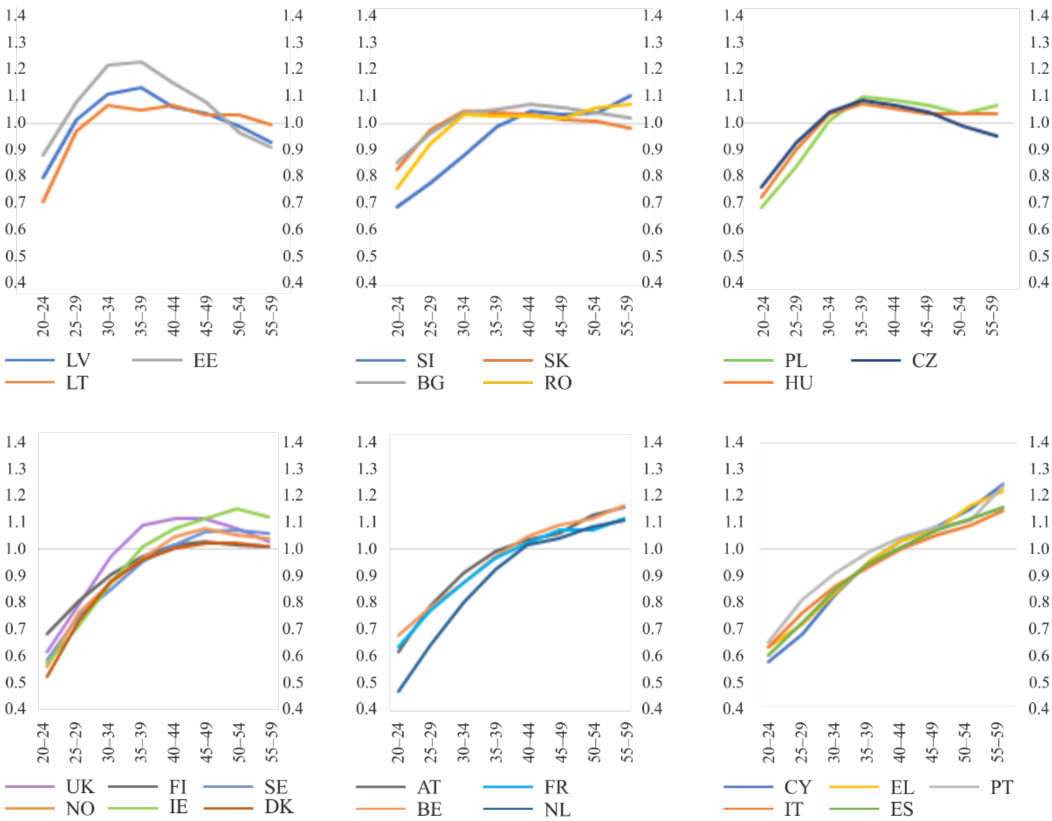
The remainder of the paper is organized as follows: Section 2 provides some evidence for the existence of the characteristics that affect the net wealth distribution, such as the productivity gap among generations and demographic profiles of countries. We also explore the responses of consumption to the monetary policy shock for different age groups in Latvia (Eastern EU) and Italy (Western EU), using micro level data. Section 3 sets up the model, which incorporates the multi-period OLG model into the basic New Keynesian framework of Clarida et al. (1999). Section 4 outlines the calibration for the quantitative exercise. In Section 5, we conduct experiments with the introduced theoretical model by employing economies with different demographic structures and productivity levels. Then we interpret and compare the responses of these economies to an unexpected monetary policy shock. Finally, Section 6 summarizes and concludes.

2. EMPIRICAL EVIDENCE

In this section, first, we provide empirical evidence for the existence of the characteristics that affect the shape of net wealth distribution, such as the productivity gap among generations and demographic profile of countries. Next, in order to support our theoretical findings, we explore the responses of consumption to a monetary policy shock for different age groups in Latvia (Eastern EU) and Italy (Western EU), using household consumption data aggregated by the age of the household reference person.

There are several characteristics of economies that could explain the cross-country differences in the shape of net wealth distribution by age. Among these factors, we observe a significant difference in the age structure and the historical backgrounds of these two groups of countries. According to Eurostat, the OAD ratio, i.e. the ratio of population aged 65+ to that aged 20–64, grew in the 28 EU countries over the last 38 years (from around 20% in 1970 to 33% in 2018 (see Figure B.1). This raises a question about the impact of aging population on the effectiveness of monetary policy in the EU. Kantur (2013) shows that in older economies the effectiveness of monetary policy is lower due to the decreasing interest rate sensitivity of the whole economy. The OAD ratio of EU countries is reported in Figure B.1. We observe that Eastern EU countries are relatively younger than the Western ones, implying a higher population growth rate. As mentioned above, Eastern EU countries were part of the centrally planned economic system until the end of the 20th century, and the collapse of the old system led to discrepancies in labour productivity between generations in these countries. A paper related to this issue has been co-authored by Lovász and Rigó (2013). It proves the existence of the old-young productivity gap, using linked employer-employee data for Hungary for the period 1986–2008. During this period, Hungary, similarly to other Eastern EU countries, underwent a rapid economic transition. Therefore, access to the new technologies and resources for workers of various age groups could vary and result in different wage levels. According to the findings of the paper, in 1992–1995, older skilled workers in Hungary became relatively less productive than the young workers. The resulting productivity gap between generations became statistically insignificant in 2008. Throughout the paper, we refer to the situation when younger workers are more productive than the older ones at a given time period, i.e. the productivity gap between generations, as *generation heterogeneity*. In Eastern EU countries, generation heterogeneity should result in higher labour income of younger workers. In order to check this, we employ EU-SILC[3] micro data for the period 2005–2017.

Figure 2
Wage ratios by age and country



Source: EU-SILC micro data, Eurostat. Data period: 2005–2017, for Bulgaria and Romania 2007–2017. Note. For each age category and country, the Figure displays the ratio between the (full-time) wage level of the particular age group and the mean (full-time) wage level of the country. Wages are normalized for each sector, year and country. The ratio equalling one represents the average wage level in the country.

Figure 2 presents wage-age profiles for 25 EU countries. For each year and country, the wage levels are normalized across sectors. For each age category, a data point represents the weighted average of ratios between the individual wage and mean wage for the whole country/year over the period 2005–2017. The ratio equalling one implies that wages correspond to the country's mean wage in the particular age category. In the majority of Western EU countries, the ratio of an individual wage to the mean one increases gradually and reaches the mean level after the age of 40 (except the UK where the age-wage profile is more hump-shaped, and it is more in line with the age-

[3] European Union Statistics on Income and Living Conditions.

wage profile of the US (see Lagakos et al. (2018)). However, in Eastern EU countries, particularly in Latvia, Lithuania and Estonia, the age-wage profiles are more hump-shaped, indicating the country's mean wage level for young workers, and below the country's mean wage level for older workers. It is worth emphasizing that Figure 2 describes the cross-sectional aspect of the wage-age profile of workers. Therefore, it does not necessarily entail that the life-time wage schedule of an individual worker in Eastern EU countries follows the hump-shape during his/her life. Furthermore, according to the official statistics, there is a significant difference in the average annual real labour productivity growth per person between Eastern and Western EU countries. During the last 18 years, real labour productivity increased annually by 3% in Eastern EU countries and by 1% in Western EU countries (see Figure B.2). Assuming that there is *generation heterogeneity* between cohorts, the shape of age-wage productivity profiles of Western EU countries should be more flat comparing to Eastern EU countries which is in line with the wage-age profile presented in Figure 2.

We also estimate the BVAR in order to investigate the responses of consumption to the monetary policy shock for different age groups in Latvia (Eastern EU) and Italy (Western EU). We use data on the growth rate of food consumption in Italy and Latvia for households with young (20–39), middle-aged (40–59) and old (60+) main reference person. The employed consumption data have been obtained from two different data sources: for Latvia – from the Household Budget Survey (2002–2016) and micro data for Italy – from the Survey on Household Income and Wealth (1996–2016) (see Appendix C, Table C.1 and Figure C.2 for a detailed description of the dataset). The three-variable BVAR approach is used: the growth rate of food consumption, 3-month shortterm interest rate and inflation. We estimate separate BVARs for each age category and country, allowing for two lags. The Normal-Wishart prior distribution with standard tightness parameters (overall tightness $\lambda = 0.2$, cross-variable weighting $\lambda_2 = 0.5$, lag decay $\lambda_3 = 2$, and exogenous variable tightness $\lambda_4 = 100$) and 4000 iterations are used. Sign restrictions are employed to separate monetary policy disturbances from other macroeconomic shocks, such as aggregate demand and supply (see Table 1). Due to annual frequency of data, sign restrictions are imposed on impact. For identification of the monetary policy shock, we followed the conventional outcome of tightening monetary policy which suggests reduction in both inflation and output (Georgiadis (2015), Feldkircher and Huber (2016), Chen et al. (2017)). For the purposes of this paper, we use food consumption of different age cohorts instead of output.

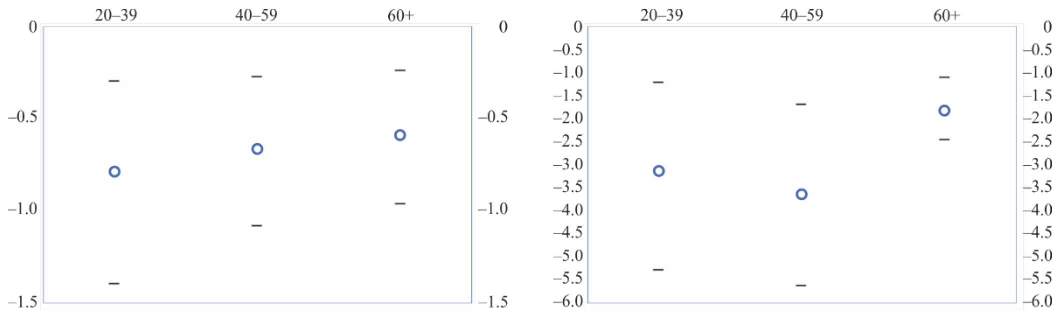
Table 1
Sign restrictions

	Inflation	**3-month interest rate**	**Consumption**
Monetary policy	–	+	–
Aggregate supply	+	+	–
Aggregate demand	–	–	–

Note. Due to the annual nature of data, all restrictions are imposed on impact.

Figure 3 presents the impulse response of food consumption (during the first year) to an increase in interest rate by 25 bps. First, we observe that there are significant differences in the responses of Italy and Latvia, with more pronounced responses in the latter. Second, we observe that older households respond less to a change in the interest rate. This result can be interpreted as weakening of the *substitution effect* and the strengthening of the *wealth effect* by age. Similarly, for the US, Wong (2016) and Berg et al. (2019) investigate the same question and find that the responses are higher for the young people. Moreover, Berg et al. (2019) suggests that in the US the response of the consumption of older households could be positive to a tightening monetary policy shock, pointing to a very strong *wealth effect*.

Figure 3
Change in food consumption (in the first year) to an increase in interest rate by 25 bps



(a) Italy, food consumption (b) Latvia, food consumption

Note. The figure displays the median impulse responses and the 68% confidence bands based on 4000 iterations.

3. MODEL

This section introduces a model, which incorporates a T-period OLG setup to the standard New Keynesian framework). The OLG setup enables us to introduce aging and generation heterogeneity to the model, and the DNK framework has the convenient environment to study the effectiveness of monetary policy.

The demand side of the economy, the households' problem, is modeled as an OLG setup introduced by Samuelson (1958) and Diamond (1965). Agents have finite lifetimes, even though the economy lasts forever. All agents in the economy are born as workers. In the first R period (R is the retirement age) of their lifetime, agents earn wage income by supplying labour and decide how much to consume and save. Agents can use two types of assets to make savings: one-period nominally riskless discount bonds yielding a nominal return and equity shares of the firms which are infinite-lived assets. It is crucial to have a stock market in this setup because it links the short-lived agents to infinitely lived firms. The ownership of the firms is transferred through the equity market. All agents retire after R period of working. In the retirement period, retirees stop supplying labour but continue to save in bonds and equity and consume from their wealth, and at the end of period T they die. The supply side of the economy has the basic New Keynesian framework a lá Clarida et al. (1999). Monetary policy follows a Taylor (1993) rule where the central bank reacts to the output and inflation.

Following the aforementioned discussion on the distribution of labour income between cohorts at a given time period, we have modified the demand side of the model by introducing *generation heterogeneity*. In Eastern EU countries, due to the collapse of the former economic system, older agents of today are less productive than younger generations. Over time, the productivity gap between generations will disappear and resemble the case in Western EU countries. However, for now we observe differences in labour income and productivity levels between cohorts. To capture this fact, we assume that productivity levels of generations are constant over their life time however the generation productivity grows by g from generation to generation.

3.1 Demographics

The time period of the model is discrete. During each one-year period, the household sector consists of T overlapping cohorts aged between 0 and T. In this model, we use a representative household j of a representative generation j' for each period t.[4] In each period t, a new generation is born into the economy, and the existing generations become one period older. For the sake of simplicity, we assume that only the first cohort of the generation is fertile. The exogenous constant population growth rate of the new cohort in period t is denoted by n. Therefore, the population grows by n in each period. The number of retired agents at time t is:

$$N_{t+k}^{r} = (1+n)^{k} \sum_{i=0}^{T-R-1} (1+n)^{i}.$$

The number of workers at time t is

$$N_{t+k}^{w} = (1+n)^{k} \sum_{i=T-R}^{T-1} (1+n)^{i}.$$

Finally, it is useful to define an indicator for aging, which is the old-age dependency ratio denoted by OAD. It is the ratio of the retired to employed agents in period t. The OAD ratio decreases as the population growth rate increases. This study assumes that population growth rates are constant over time but different across the economies. In other words, the OAD ratio does not change across time in either economy.

$$OAD = \frac{N_t^r}{N_t^w}.$$

3.2 Households

In the life-cycle economy employed in this model, all agents are born as workers. In the first R-period of their lifetime, agents earn wage income by supplying labour and decide how much to consume and save. Agents can use two types of assets for making savings: one-period nominally riskless discount bonds yielding a nominal return and equity shares of firms which are infinite-lived assets. All agents retire after R period of working. In the retirement period, retirees stop supplying labour but continue to save in bonds and equity and consume from their wealth. At the end of their lifetime, they consume everything and die. We assume that agents do not leave bequests to their offspring.

The life-time utility function of a representative household j born in period t who is a member of generation j' is given by:

[4] Generation j' was born at time t and lives until $t+T$.

$$V_t^{j,j\prime} = \sum_{k=1}^{R} \beta^{k-1} E_{t+k} \left(\frac{C_{t+k-1}^{k}(j)^{1-\sigma}}{1-\sigma} - \frac{N_{t+k-1}^{k}(j)^{1+\psi}}{1+\psi} \right) + \sum_{k\prime=R+1}^{T} \beta^{k\prime-1} E_{t+k\prime} \left(\frac{C_{t+k\prime}^{k\prime}(j)^{1-\sigma}}{1-\sigma} \right)$$

where E is the expectations operator, β is the individuals' time discount factor. The parameters σ and ψ represent the inverse of intertemporal elasticity of substitution and labour supply respectively. C^k and N^k are the consumption and labour supply decision of cohort k of generation j'. The household j earns wage income $WZN^k(j)$ until period R where Z denotes the productivity of the agent.

In addition to labour income, agents earn financial income from their savings made in the previous period. The savings occur in the form of equity shares of firms or one-period nominally riskless discount bonds. Agents can continue to save until the age $T-1$ and get return from savings until the age T. In the last period of their lifetime, the representative retiree j consumes all his leftover wealth and then dies. Formally, the budget constraints that the representative household j of generation j' faces are:

$$P_t C_t^1(j) + B_t^1(j) + P_t \int_0^1 Q_t(i) S_t^1(i,j) di = W_t Z_t N_t^1(j),$$

$$P_{t+1} C_{t+1}^2(j) + B_{t+1}^2(j) + P_{t+1} \int_0^1 Q_{t+1}(i) S_{t+1}^2(i,j) di = W_{t+1} Z_t N_{t+1}^2(j),$$

$$+B_t^1(j)(1+i_t) + P_{t+1} \int_0^1 \big(Div_{t+1}(i) + Q_{t+1}(i)\big) S_t^1(i,j) di,$$

$$\vdots$$

$$P_{t+T} C_{t+T}^T(j) = B_{t+T-1}^{T-1}(j)(1+i_{t+T-1}) + P_{t+T} \int_0^1 (Div_{t+T}(i) + Q_{t+T}(i)) S_{t+T-1}^{T-1}(i,j) di$$

where $B^n(j)$ represents nominal bond holdings of agent j and i refers to the nominal interest rate. P refers to the price of a consumption good. $Div(i)$ and $Q(i)$ represent real dividends paid by the monopolistically competitive firm i and the price of a share of the firm i respectively. $S_t(i,j)$ shows the amount of shares of firm i held by household j. Unlike the standard DNK model, we have an equity market in this setup, which enables us to combine the short-lived agents to infinite living firms. The ownership of the firm is transferred through the equity market, that is to say, when agents buy stocks of firms and become their owners. Finally, Z_t denotes the productivity level of an agent who was born at time t and belongs to the generation j'. We assume that the productivity level of an agent remains constant over lifetime. This means that if you were born at time t with productivity level Z_t, it stays as Z_t until the retirement age. However, the productivity level is growing by g percent from generation to generation. Therefore, an agent, who was born at time $t+1$ (who belongs to generation $j'+1$), is g percent more productive compared to the previous generation j'. This assumption leads to a *heterogeneity among generations* in terms of productivity levels in the model.

The intertemporal budget constraint of the representative agent of generation j' is:

$$\sum_{k=0}^{T-1} \frac{C_{t+k}^{k+1}}{\prod_{j=0}^{k}(1+r_{t+j-1})} = Z_t \sum_{k=0}^{R-1} \frac{W_{t+k}}{P_{t+k}} \frac{N_{t+k}^{k+1}}{\prod_{j=0}^{k}(1+r_{t+j-1})}.$$

The first order conditions of the household's problem are:

$$C_t^k(j)^{-\sigma} \frac{W_t Z_{j'}}{P_t} = N_t^k(j)^{\psi} \quad k \in (1, R) \qquad (1),$$

$$C_t^k(j)^{-\sigma} = \beta E_t \left\{(1+i_t)\frac{P_t}{P_{t+1}} C_{t+1}^{k+1}(j)^{-\sigma}\right\} \quad k \in (1, T-1). \qquad (2),$$

$$C_t^k(j)^{-\sigma} = \beta E_t \left\{\left[\frac{Q_{t+1}(i)+Div_{t+1}(i)}{Q_t(i)}\right] C_{t+1}^{k+1}(j)^{-\sigma}\right\} \quad k \in (1, T-1) \qquad (3),$$

$$\Lambda_{t,t+1} = \frac{1}{(1+i_t)} = \beta E_t \left\{\left(\frac{C_{t+1}^{k+1}}{C_t^k}\right)^{-\sigma} \frac{P_t}{P_{t+1}}\right\} \qquad (4).$$

Rearranging equations (2) and (3), we get the no arbitrage condition:

$$Q_t(i) = \frac{1}{1+r_t}[Q_{t+1}(i) + Div_{t+1}(i)],$$

which suggests that it does not matter whether workers save in riskless bonds or stocks in the equity market. Both yield the same real return, $r_t = i_t - E_t\pi_{t+1}$, in such a riskless economy.

3.3 Firm side

The supply side of the economy is modeled based on the basic New Keynesian framework following Clarida et al. (1999). There are two types of firms: producers of consumer goods and producers of intermediate goods. There is imperfect competition in the intermediate goods market due to the assumption that each firm produces a differentiated good. We follow a staggered price setting a lá Calvo (1983), in which a random fraction of firms optimally sets prices in each period.

3.3.1 Consumption (final) goods producers

There is a continuum of intermediate goods indexed by $i \in [0,1]$, which are transformed into a homogenous consumption good according to the constant returns to scale production function

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}}$$

where $Y_t(i)$ is the quantity of the intermediate good i, and $\varepsilon > 1$ denotes the price elasticity of demand. The consumption goods sector is subject to perfect competition, which determines the demand function for the representative intermediate good i

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} Y_t$$

where $P_t(i)$ and P_t denote the price of good i and the average price level respectively. Reflecting the CES-structure of the technology in the final goods sector, P_t is given by

$$P_t = \left(\int_0^1 P_t(i)^{1-\varepsilon} di\right)^{\frac{1}{1-\varepsilon}}.$$

3.3.2 Intermediate goods firms

In the intermediate goods sector, there is a continuum of firms indexed by $i \in [0,1]$. Each firm produces a differentiated good i with the production function. All the firms have identical technology represented by the following production function at time t:

$$Y_t(i) = Z_t \left[\sum_{k=1}^{R} \frac{N_t^k(i)}{(1+g)^{k-1}}\right] \quad (5)$$

where $Y_t(i)$ and $N_t^k(i)$, respectively, denote the output of firm i and the hours worked required by firm i from the cohort k at time t. The generation productivity level grows by $Z_t = (1+g)Z_{t-1}$.[5] The productivity-adjusted output level is $\mathcal{Y}_t = Y_t/Z_t$, and the productivity-adjusted production function is:

$$\mathcal{Y}_t(i) = \left[\sum_{k=1}^{R} \frac{N_t^k(i)}{(1+g)^{k-1}}\right] \quad (6).$$

The labour market is competitive, i.e. the nominal wage rate W is taken as given in the production of good i. Intermediate firms are owned by equity holders and are managed to maximize the profit of the current owners. In the final goods production sector, the intermediate firm i faces a downward sloping demand curve. At time t, real profits (dividends) are:

$$Div_t(i) = \mathcal{Y}_t(i) - \frac{W_t}{P_t} N_t(i).$$

$N_t(i) = \sum_{k=1}^{R} \frac{N_t^k(i)}{(1+g)^{k-1}}$ is the labour demand of an intermediate firm i. Under flexible prices, after witnessing the shock, firms choose price P_t^*,

$$P_t^* = \frac{\varepsilon}{\varepsilon-1} \frac{W_t}{P_t} P_t = \mathcal{M} MC_t$$

where MC_t and $\frac{\varepsilon}{\varepsilon-1} = \mathcal{M}$ denote the marginal cost and the desired markup value respectively. In a symmetric equilibrium, $P_t^* = P_t$ and $\frac{W_t}{P_t} = \frac{1}{\mathcal{M}}$.

Following Calvo (1983), the nominal price rigidity is modeled by allowing random intervals between price changes. At each period, a firm adjusts its price with a constant probability $(1-\theta)$ and keeps its price fixed with probability θ. The reoptimizing firm solves

$$\max_{P_t^*} \sum_{k=0}^{\infty} \theta^k E_t\{\Lambda_{t,t+k}(P_t^* \mathcal{Y}_{t+k}(i) - W_{t+k}\mathcal{Y}_{t+k}(i))\}$$

[5] We normalize the variables by dividing them by the productivity level of the youngest cohort at a given time period.

subject to the demand function of the intermediate good. The first order condition is:

$$\sum_{k=0}^{\infty} \theta^k E_t \left\{ \Lambda_{t,t+k} \mathcal{Y}_{t+k}(i) \left(P_t^* - \frac{\varepsilon}{\varepsilon - 1} W_{t+k} \right) \right\} = 0$$

where $\Lambda_{t,t+1}$ is the stochastic discount factor formalized in equation (4).

The owners of reoptimizing firms set their prices according to the above optimality condition, considering the expected profit of the firm. The existence of the equity market is crucial in this setup. If we do not have the equity market, the owner of the firm will just maximize the profit of the current period because he/she dies at the end of the period. However, the equity market enables us to utilize the standard firm side problem as in the DNK setup. Following the no arbitrage condition, firm owners should maximize the expected profit of the firm to set the maximum price of the stocks today.

3.4 Central bank

The monetary policy authority follows a standard Taylor (1993) type feedback rule:

$$i_t = \rho\ i_{t-1} + (1 - \rho)[\phi_\pi(\pi_t) + \phi_y(\hat{y}_t)] + v_t$$

where ϕ_π and ϕ_y are feedback parameters, π_t is the deviation of rate of inflation from its steady state value and $\hat{y}_t$ is the deviation of the level of productivity-adjusted output from its steady state value. The parameter ρ denotes the degree of policy inertia. The exogenous component of the monetary policy is denoted by v_t and follows an AR(1) process

$$v_t = \rho_v v_{t-1} + \varepsilon_t^v$$

where ε_t^v denotes the monetary policy shock and $\rho_v \in [0,1)$ shows the persistence of the shock.

3.5 Market clearing and equilibrium conditions

This section presents the market clearing conditions for the model economy. It is worth emphasizing that the per worker and per capita variables are the same in the standard DNK setup aggregate. However, in this analysis they are all different and have to be kept track of. The variables are normalized by youngest agents' productivity level Z and the number of workers at a given time period N^w at time t. The goods market clearing condition requires

$$\mathcal{Y}_t = \textstyle\sum_{k=1}^{T} \mathcal{C}_t^k \tag{7}$$

where $\mathcal{Y}_t$ and $\mathcal{C}_t^k$ refer to the productivity-adjusted per worker output level and productivity-adjusted per worker consumption level of cohort k. Labour market clearing implies

$$N_t = \textstyle\sum_{k=1}^{R} \frac{N_t^k(i)}{(1+g)^{k-1}} = \mathcal{Y}_t \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} di \tag{8}$$

where the term $\left(\int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} di\right)$ is the measure of price dispersion across firms. At equilibrium, agents do not trade bonds among themselves therefore total bond holdings are $\sum_{k=1}^{T-1} B_t^k = B_t = 0$. The aggregate stock outstanding equity for each intermediate goods producing firm must equal the corresponding total amount of the issued shares normalized to 1 $\forall i \in [0,1]$. Hence, the market clearing condition for shares at time t requires $\sum_{j=1}^{T-1} S_t(i,j) = 1$.

Finally, productivity-adjusted real dividend payments by intermediate firms and real stock price index are given:

$$Div_t = \int_0^1 Div_t(i)di \qquad Q_t = \int_0^1 Q_t(i)di \tag{9}$$

In order to analyse the dynamics of the model, we start by deriving the log-linearized equilibrium conditions.

3.6 Log-linearized dynamics

This section provides the log-linearized equations around the zero inflation steady state. We use lower case letters to show the log of the variable and the hat to indicate the percentage deviation from its steady state value. The demand side equations are as follows. The labour supply decision of the representative agent:

$$\widehat{w}_t - \hat{p}_t = \psi \hat{n}_t^k + \sigma \hat{c}_t^k \quad k \in (1, R) \tag{10}$$

where $\hat{c}_t^k = log(C_t^k/(C^{k*}ZN_t^w))$ denote log deviation of consumption of cohort k from its value along the balanced growth path.

The Euler equation:

$$\hat{c}_t^k = E_t\{\hat{c}_{t+1}^{k+1}\} - \frac{1}{\sigma}[\hat{\imath}_t - E_t\{\hat{\pi}_{t+1}\}] \quad k \in (1, T-1) \tag{11}$$

Equation (10) denotes the labour supply decision of cohort k at time t. Equation (11) is the linear Euler equation of agent at time t. Unlike the standard New Keynesian model with a representative agent, the individual agent's Euler equation and the goods market condition are not sufficient to derive the dynamic IS equation. The derivation of IS equation needs further work by aggregating the individual consumption functions obtained by combining the Euler equation and intertemporal budget constraint of the agent. The OLG–IS equation not only depends on the current period's interest rate and the expected inflation but also on the historical interest rate, the expected inflation rate and the realized inflation. This is due to the fact that at time t there are T different types of agents, optimizing according to the available information. Young agents at time t decide on their consumption, using information about the current period. However, older agents at time t have chosen their consumption levels in the previous periods, using the information available at that time. Therefore, compared to the standard DNK–IS equation, a richer dynamic system is achieved. The OLG–IS equation at time t is:

$$\hat{y}_t = \sum_{k=1}^{T} \frac{C_k^*}{y^*} \hat{c}_t^k \tag{12}$$

where $\hat{y}_t$ denotes log deviation of productivity-adjusted per worker output from its value along the balanced growth path. The slope of the OLG–IS equation depends on the distribution of weights of consumption levels of cohorts. The distribution of weights depends on the population growth rate, n, and the growth rate of productivity level among generations, g.

Finally, equations (13) and (14) show the log-linear form of stock price and dividend equations.

$$\hat{q}_t = \Omega[(\frac{div^*}{q^*}\widehat{div}_{t+1} + \hat{q}_{t+1}] - (\hat{\imath}_t - E_t\{\pi_{t+1}\})) \tag{13}$$

$$\widehat{div}_t = \frac{y^*}{div^*}\hat{y}_t - \frac{W^*N^*}{P^*div^*}(\widehat{w}_t - \hat{p}_t + \hat{n}_t) \tag{14}$$

where $\Omega = \frac{(1+g)(1+n)}{(1+i^*)}$.

The log-linearized equations of the supply side of the model are the production function and the forward-looking Phillips equation.[6]

$$\hat{y}_t = \hat{n}_t \qquad (15),$$

$$\pi_t = \tilde{\beta}\widetilde{\Lambda}E_t\{\pi_{t+1}\} + \tilde{\kappa}\widehat{mc}_t \tag{16}$$

where $\widehat{mc}_t = \widehat{w}_t - \hat{p}_t$ is deviation of the real marginal cost from its steady state and $\tilde{\kappa} = \frac{(1-\theta)(1-\theta\phi)}{\theta}$. $\widetilde{\Lambda} = (C_t^{k+1}/C_t^k)^{-\sigma}$ is the steady state of (gross) growth rate of consumption. We can express the inflation as the discounted sum of current and expected future deviations of marginal costs from steady state by solving the above equation forward.

Due to the OLG setup in the demand side, we have an unconventional Phillips equation. Unlike the standard Phillips equation, both weights of the expected inflation and the marginal cost depend on the productivity and the population growth rates. The finite lifetime of agents, compared to the standard DNK framework, leads society to understate the expected inflation. However, as population ages (and/or the productivity growth rate increases), both coefficients, ϕ and $\tilde{\kappa}$, converge to the values of the infinitely-lived agent problem.

The equilibrium is characterized by equations (10)–(16), together with a description of monetary policy.

3.7 Balanced growth path

To obtain the values of labour supply N_k^* $k \in (1, R)$, consumption C_k^* $k \in (1, T)$ and interest rate i^* on *balanced growth path* (BGP), we solve the system of equations consisting of $T-1$ steady state Euler equations, R steady state intratemporal labour-consumption equations, the intertemporal budget constraint and the market clearing condition for a given set of parameters σ, ψ, β, ε as well as population and productivity growth rates n and g.

The variables on BGP are normalized by the productivity of the youngest cohort and the number of workers at a given period.

$$C_{k-1}^{*-\sigma} = \beta((1+n)(1+g))^{-\sigma}C_k^{*-\sigma}(1+i^*) \qquad k \in (2, T),$$

$$C_k^{*-\sigma}\frac{1/\mathcal{M}}{(1+g)^{k-1}} = N_k^{*\psi}\left(\frac{\sum_{i=0}^{R-1}(1+n)^i}{(1+n)}\right)^{\sigma+\psi} \qquad k \in (1, R),$$

[6] See equation (7) for a detailed derivation of the supply-side equations and the Phillips relation.

$$\sum_{k=1}^{T} C_k^* \left(\frac{(1+n)(1+g)}{1+i^*}\right)^{k-1} = \frac{1}{\mathcal{M}} \sum_{k=1}^{R} N_k^* \left(\frac{1+n}{1+i^*}\right)^{k-1},$$

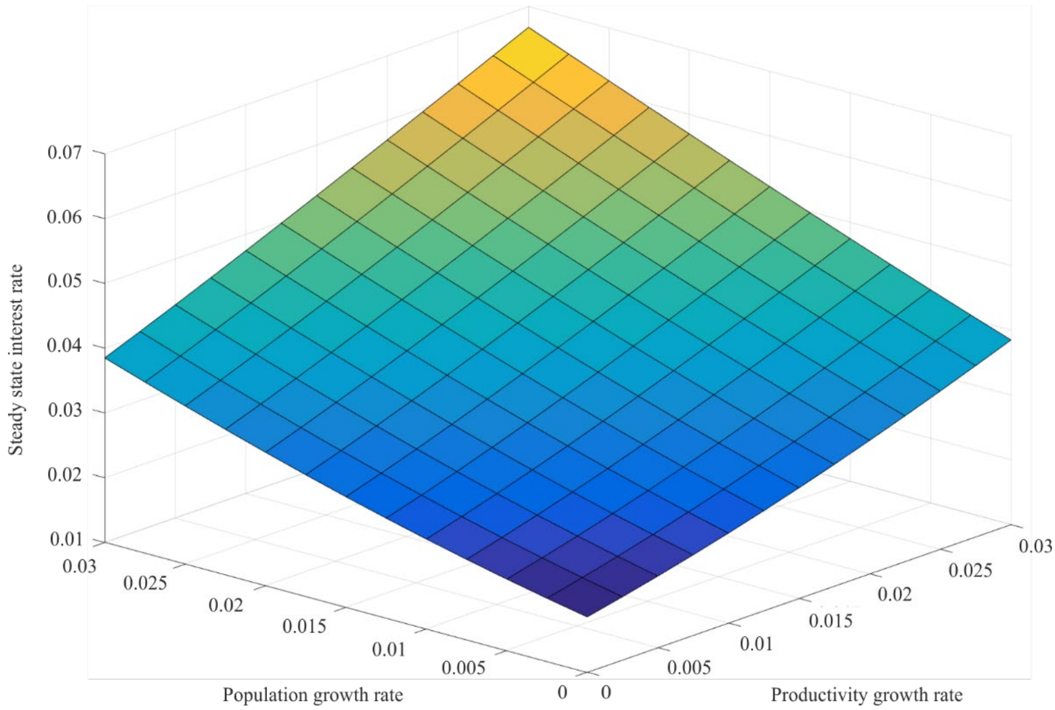
$\sum_{k=1}^{T} C_k^* = \sum_{k=1}^{R} \frac{N_k^*}{(1+g)^{k-1}}.$

Since the growth rate of consumption is constant, the discount factor implied by (4) on the BGP is also constant:

$$\Lambda = \frac{1}{1+i^*} = \beta[(1+n)(1+g)]^{-\sigma} \left(\frac{C_{k+1}^*}{C_k^*}\right)^{-\sigma}.$$

The above relation suggests that the steady state interest rate increases along with the population growth rate, other parameters being equal. This relation is consistent with the empirical literature[7] and captures the fact that a decrease in the number of workers implies lower labour compared to capital which is similar to financial assets in our framework, and this leads to a decrease in the interest rate. A similar relation is observed between the productivity growth rate among generations and the steady state interest rate. If the younger generation is more productive compared to the previous generation, the steady state interest rate rises. When the productivity growth is high, agents expect their future income to be higher than their current one. Therefore, to smooth out their consumption, they save less today, and this leads to higher interest rates. Figure 4 illustrates the relation between productivity and the population growth rate with the value of steady state interest rate on the BGP.

Figure 4
The relation between steady state interest rate and growth rates of productivity and population
($\sigma = 1$, $\psi = 1$, $\beta = 0.98$)



[7] Krugman (1998) points out that the aging population of Japan suppressed the natural rate of interest.

Using steady state values of consumption C^*, hours worked N^*, and interest rate i^*, we calculate the value of (productivity-adjusted) stock prices, q^*, and dividends, div^*, on the BGP.

$$div^* = (1 - 1/\mathcal{M}) \sum_{k=1}^{T} C_k^*,$$

$$q^* = \frac{(1+n)(1+g)}{i^* - n - n\,g - g} div^*.$$

Moreover, by using the above steady states and period budget constraints, we calculate the wealth accumulation of an agent over lifetime. In our analysis, wealth of a cohort is the summation of riskless bond and equity holdings of an agent and represented as: $A_k^* = B_k^* + q^* S_k^*$.

$$A_1^* = \left(\frac{1}{\mathcal{M}} N_1^* - C_1^*\right),$$

$$A_k^* = \left(\frac{1}{\mathcal{M}} \frac{N_k^*}{(1+g)^{k-1}} - C_k^* + \frac{(1+i^*)}{(1+g)(1+n)} A_{k-1}^*\right) \quad k \in (2, R),$$

$$A_k^* = \left(-C_k^* + \frac{(1+i^*)}{(1+g)(1+n)} A_{k-1}^*\right) \quad k \in (R+1, T-1).$$

Values of A^* represent the distribution of lifetime wealth of an individual on the BGP. Meanwhile, it also shows the distribution of wealth among cohorts.

4. CALIBRATION

In this section, we outline the parameterization for the quantitative analysis. The model is calibrated to annual frequency. Table 2 summarizes the parameter and steady state values of key variables. In models (1)–(3), we set the growth rate of population n to zero and assign different values, 0%, 1% and 2%, for the productivity growth rate g between cohorts. In models (4)–(6), different from models (1)–(3), the population growth rate is set to 0.25%, implying the economy with lower OAD ratio. For plausible model comparison, we keep parameters of (inverse of) intertemporal elasticity of substitution σ, elasticity of labour supply ψ, price elasticity of demand (and therefore markup $\mathcal{M}$) and the (subjective) discount factor β constant (see Table 2). The population and productivity growth rate parameters are chosen to replicate the OAD ratios, the shape of net wealth distribution and the change in labour supply and consumption between cohorts observed in the HFCS data for Eastern and Western EU countries. We also compare the obtained ratios of gross income-to-consumption, net wealth-to-consumption and net wealth-to-income to the ratios acquired from the second wave of HFCS.[8]

The average life expectancy in Europe is approximately 85 years. We set T to 65, assuming that people start working at the age of 20 and die at the age of 85. According to the Eurostat statistics, the OAD ratio in 28 EU countries over the last four decades has been growing from 20% in 1980 to 33% in 2018. Italy is the oldest European country with the average OAD ratio of 33% in 2000–2018 (see Appendix B and Figure B.1). During the same period, the OAD ratio of Eastern and Western European countries was around 27%. For the calibration purposes, we choose population growth

[8] The second wave of HFCS took place in 2013–2014. Details on the year of reference for each country can be found in HFCS (2016), Table 1.1.

rate to fit the OAD ratio for the selected country groups (Western EU, Eastern EU and Italy[9]).

Table 2
Parameter values and steady states

	(1)	(2)	(3)	(4)	(5)	(6)
σ	1	1	1	1	1	1
ψ	1	1	1	1	1	1
n	0.0	0.0	0.0	0.0025	0.0025	0.0025
g	0.0	0.01	0.02	0.0	0.01	0.02
ε	11	11	11	11	11	11
μ	1.1	1.1	1.1	1.1	1.1	1.1
β	0.98	0.98	0.98	0.98	0.98	0.98
$div*$	0.098	0.077	0.062	0.098	0.077	0.063
$q*$	5.30	5.57	5.95	5.68	6.02	6.42
$i*$	0.019	0.024	0.031	0.020	0.025	0.033
OAD	30.0	30.0	30.0	27.6	27.6	27.6

Source: authors' estimations for different model specifications.
Note. T = 65, R = 50.

During 1996–2018, the average annual growth rate of labour productivity was positive and below 1% in most of Western EU countries, except a 0% average annual growth rate in Italy. On the other hand, in Eastern EU countries the average growth rate of labour productivity was above 3% during the corresponding period (see Appendix B and Figure B.2). The observed difference in productivity growth rates is embedded in our model as the assumption of the *generation heterogeneity*. We assume that the productivity level of the agents remains the same throughout their lifetime. However, the productivity level increases by g percent from generation to generation. Therefore, at a given point in time, the productivity level of workers declines with age.

Table 3
Net wealth distribution by country group

	Italy	Western EU	Eastern EU
Age of mean net wealth level	55–60	40–45	35–40
Age of maximum net wealth level	70–75	65–70	50–55

Source: authors' estimations using HFCS wave 2 data.
Notes. Net wealth variables are estimated per adult member of a household; age reflects the age of a household's reference person. Eastern EU countries are Hungary, Latvia, Estonia, Slovenia and Slovakia. Western EU countries are Austria, Belgium, Cyprus, Germany, Spain, Finland, France, Greece, Ireland, Italy, Luxembourg, Malta, the Netherlands and Portugal. Value by region is estimated as simple average.

[9] We separate the Italian case due to the unique combination of two key model parameters – both population and productivity growth rates are close to zero.

Table 4
Model calibration results – the shape of net wealth distribution

	(1)	(2)	(3)	(4)	(5)	(6)
Age of mean net wealth	50	41	33	48	38	31
Age of maximum net wealth	69	69	52	69	69	49

Source: authors' estimations for different model specifications.

The assumption of a generation-specific productivity level allows us to fit the age of maximum and mean net wealth holdings to the results of the second wave of HFCS (see Tables 3 and 4). Figure D.1 displays the distribution of net wealth by age measured by the corresponding population and productivity growth rates defined in the models (see Table 2). As the productivity gap between generations increases and the OAD ratio decreases, the wealth distribution skews towards younger cohorts.

Furthermore, we use HFCS data to calibrate our model with respect to the growth rates of income and consumption by cohort (see Tables 5 and 6). In the model, the growth rate of consumption and labour income is constant between cohorts. According to HFCS data, the growth rate of consumption and labour income is constant for the age group 35–75, since the maximum level of income is already achieved by the age of 40 in the majority of countries. In the three groups of countries, growth rates differ significantly – the slowest change between cohorts is observed in Italy, and the steepest – in Eastern EU countries (see Table 5) which is consistent with the assumption of heterogeneous productivity among generations. Higher productivity growth rate between the cohorts leads to a steeper decline in the wage income (see Table 6), consumption is affected by a change in wage income and net wealth. Faster accumulation of the net wealth in model (6) results in a slower decline in the growth rate of consumption as compared to labour income (see Table 6) which is in line with the results observed for Eastern EU countries presented in Table 5. In Western EU countries, the speed of decline in consumption and labour income is more similar.

Table 5
Consumption and labour growth rates by country group

	Italy		Western EU		Eastern EU	
Change in total income per household adult, age 35–75	mean	median	mean	median	mean	median
	0.000	–0.003	–0.004	–0.007	–0.015	–0.014
Change in total consumption per household adult, age 35–75	mean	median	mean	median	mean	median
	–0.001	–0.001	–0.003	–0.004	–0.007	–0.005

Source: authors' estimations using HFCS wave 2 data.
Notes. Total consumption and gross income are estimated per adult member of a household. Eastern EU countries are Hungary, Latvia, Estonia, Slovenia and Slovakia. Western EU countries are Austria, Belgium, Cyprus, Germany, Spain, Finland, France, Greece, Ireland, Italy, Luxembourg, Malta, the Netherlands and Portugal. Value by region is estimated as simple average.

Table 6
Model calibration results – change in labour income and consumption

	(1)	(2)	(3)	(4)	(5)	(6)
$N_R^*/N_{R-1}^* - 1$	0.002	–0.003	–0.010	–0.002	–0.007	–0.014
$C_T^*/C_{T-1}^* - 1$	–0.002	–0.007	–0.010	–0.003	–0.007	–0.010

Source: authors' estimations for different model specifications.

Table 7 summarizes the ratios between consumption, income and net wealth for different age groups obtained from the HFCS data. We observe that gross income-to-consumption ratios are of the magnitude 1–2. The ratio of net wealth-to-consumption is increasing with age. It can reach 8 for the older cohorts in Western EU countries and exceeds 20 in Eastern EU countries. The ratio of net wealth-to-gross income also increases with age, and is on average lower than the ratio of net wealth-to-consumption. The model calibration results presented in Table 8 are similar in size and direction to the ratios observed in the HFCS data.

Table 7
Ratios of consumption, net wealth and gross income by age and country group

	Italy		Western EU		Eastern EU	
Gross income-to-consumption	mean	median	mean	median	mean	median
up to 30 years	1.06	1.11	0.95	1.61	1.16	1.81
from 31 to 40 years	1.20	1.21	1.31	2.18	1.54	2.56
from 41 to 50 years	1.25	1.23	1.08	2.09	1.54	2.83
from 51 to 64 years	1.32	1.34	1.12	2.00	1.50	2.69
more than 65 years	1.29	1.28	1.01	1.34	1.67	2.25
Net wealth-to-consumption	mean	median	mean	median	mean	median
up to 30 years	4.02	0.67	1.89	1.67	3.39	2.04
from 31 to 40 years	6.08	3.15	3.48	4.20	5.48	5.80
from 41 to 50 years	8.10	5.23	3.83	5.86	7.82	10.82
from 51 to 64 years	10.62	7.50	4.99	7.47	11.00	15.81
more than 65 years	11.53	7.90	6.50	7.65	17.49	18.45
Net wealth-to-gross income	mean	median	mean	median	mean	median
up to 30 years	3.77	0.61	2.04	1.02	3.06	1.07
from 31 to 40 years	5.06	2.62	2.75	2.04	3.79	2.36
from 41 to 50 years	6.50	4.25	3.77	2.90	5.46	4.08
from 51 to 64 years	8.05	5.61	4.80	4.05	7.54	6.18
more than 65 years	8.98	6.19	6.75	5.71	10.72	8.87

Source: authors' estimations using HFCS wave 2 data.
Notes. Total consumption, gross income and net wealth variables are estimated per adult member of a household; age reflects the age of a household's reference person. Eastern EU countries are Hungary, Latvia, Estonia, Slovenia and Slovakia. Western EU countries are Austria, Belgium, Cyprus, Germany, Spain, Finland, France, Greece, Ireland, Italy, Luxembourg, Malta, the Netherlands and Portugal. Value by region is estimated as simple average.

Table 8

Model calibration results – steady state ratios of consumption, net wealth and labour income

	(1)	(2)	(3)	(4)	(5)	(6)
$w * N^*/C^*$						
up to 30 years	1.1	1.3	1.5	1.1	1.3	1.6
from 31 to 40 years	1.1	1.2	1.2	1.1	1.2	1.2
from 41 to 50 years	1.2	1.1	1.0	1.2	1.1	1.0
from 51 to 64 years	1.2	1.0	0.8	1.2	1.0	0.7
more than 65 years	1.3	1.0	0.6	1.2	0.9	0.6
A^*/C^*						
up to 30 years	0.5	1.8	3.6	0.8	2.2	4.1
from 31 to 40 years	1.9	4.9	8.5	2.5	5.7	9.5
from 41 to 50 years	3.9	7.5	11.5	4.8	8.5	12.5
from 51 to 64 years	7.7	10.4	13.1	8.3	11.1	13.7
more than 65 years	8.1	8.2	8.4	8.1	8.3	8.4
$A^*/(w * N^*)$						
up to 30 years	0.5	1.5	2.5	0.7	1.7	2.7
from 31 to 40 years	1.6	4.1	7.1	2.2	4.8	7.9
from 41 to 50 years	3.3	6.8	11.7	4.1	7.9	13.2
from 51 to 64 years	6.3	10.2	16.9	7.1	11.6	19.2
more than 65 years	9.3	13.1	20.3	10.0	14.4	23.0

Source: authors' estimations for different model specifications.

The coefficients of the Phillips relation $\phi = \tilde{\beta}\tilde{\Lambda}$ and κ (see equation (16) and Table 9) emphasize that in economies with generation heterogeneity, the current inflation depends relatively more on inflation expectations and less on deviations of the real marginal costs. As the productivity gap between cohorts declines, the relative importance of deviation in the real marginal costs increases.

Table 9.

Model calibration results – Phillips equation coefficients

	(1)	(2)	(3)	(4)	(5)	(6)
ϕ	0.982	0.986	0.990	0.983	0.987	0.990
κ	0.088	0.087	0.086	0.088	0.086	0.086
$\tilde{\beta}$	0.980	0.970	0.961	0.978	0.968	0.958

Source: authors' estimations for different model specifications.

Finally, we set the Calvo stickiness parameter, θ, to 0.75. The coefficients for Taylor rule are set to $\phi_\pi = 0.625$ for inflation and ϕ_y =0.3 for the output gap. Unlike the standard New Keynesian model, the equilibrium is seen to be determinate for the values of $\phi_\pi < 1$. This is due to the OLG setup in the demand side of the problem. The persistence of shock, ρ_v, is set to 0.2, which allows the interest rate to return to its steady state after three years. For all model specifications, we use the same Taylor rule with the same coefficient therefore accounting for the same monetary policy.

5. QUANTITATIVE ANALYSIS

In the present section, we analyse the effects of monetary policy shock on aggregate and cohort specific variables in the model economy described above. We consider the implications of innovation in the policy rate in Eastern EU (high productivity and high population growth rate) and Western EU (low productivity and low population growth rate) countries.

As discussed above, a tightening monetary policy shock has an impact on the economy via three channels: (i) the *substitution effect*: as the price of consuming today increases, agents tend to postpone their current consumption. The substitution effect gets weaker by age due to increasing marginal propensity to consume. (ii) the *wealth effect*: as the interest rate increases, future financial income of an agent rises therefore the agent tends to consume more today. The distribution of wealth is crucial for this effect. (iii) finally, since we include hours of work in the utility function, a positive shock to monetary policy leads to a change in the intertemporal labour supply decision of agents. Due to higher interest rates, agents desire to supply more labour in order to earn and save more for the future. However, the excess supply of labour dampens the wage level. Hence, labour income of workers falls and leads to a decrease in aggregate demand and labour demand which results in further decline in labour income. We call this channel the negative *income effect*. Initially, we analyse the dynamics of the model by setting the (inverse of) intertemporal elasticity of substitution, σ, to 1. This way, interest rate changes will not be transmitted to consumption through the first two channels: the *substitution effect* of an interest rate change cancels the *wealth effect* on consumption and the only channel left will be the negative *income effect*. In the sensitivity analysis section of the paper, we will analyse both individual and aggregate level responses to a positive monetary policy shock for different degrees of elasticity of substitution.

First, we compare responses of consumption and labour in Western and Eastern EU countries to a 25 bps tightening monetary policy shock for each cohort over time. Similar to the BVAR analysis provided in Section 2, we observe significant differences in the IRFs of Western and Eastern EU countries to the monetary policy shock with more pronounced responses in the latter at cohort level. Figure 5 plots the IRFs of consumption of cohorts under the assumption of 0% population and the productivity growth rate, corresponding to Italy in Western EU countries.[10] The x-axis shows the age at the time of the shock, y-axis – the years after the shock, z-axis – the deviation of consumption level from its steady state value.[11]

[10] See Figure 11 for the response of consumption and labour to 25 bps tightening monetary policy shock in Eastern EU countries.
[11] The reading of the impulse responses in this study is different from that in the standard infinite living agent models. In the standard setup, there is only one type of representative agent and the response of his/her consumption to the shock is reflected along one graph. However, in this framework, there are 65 different agents, hence the consumption and labour decisions of these agents are demonstrated in surface graphs.

Figure 5
Responses of consumption to 25 bps tightening monetary policy shock; Western EU countries

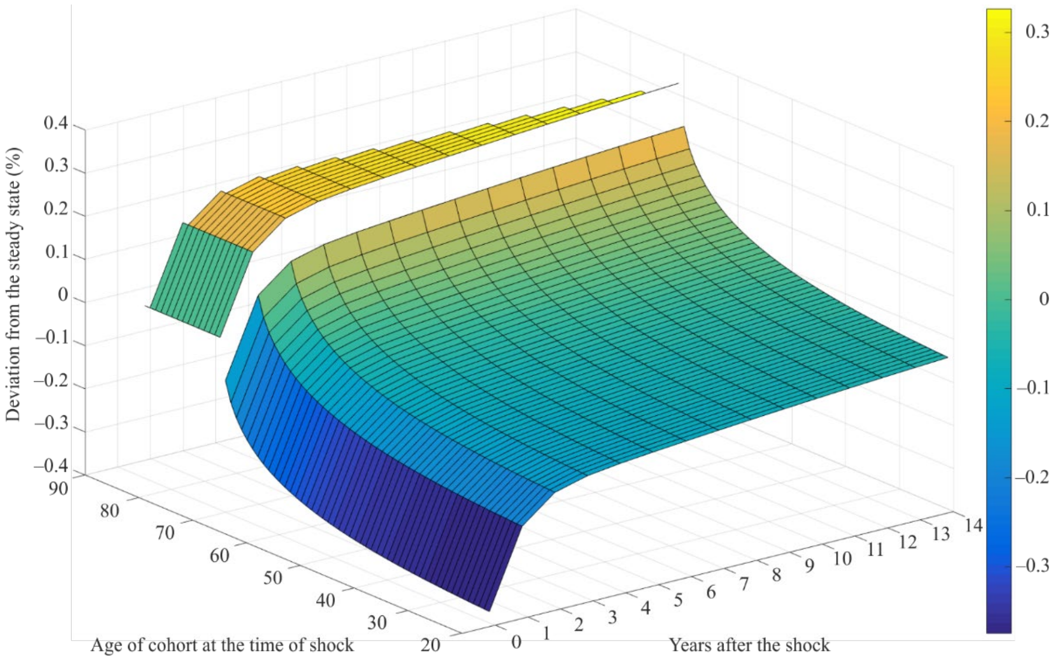


Figure 5 shows that the consumption level falls after a positive monetary policy innovation. The decline in the consumption decreases as workers get older. Moreover, the retired agents do not change their consumption since the *wealth effect* cancels the *substitution effect* under this parameterization. The negative *income effect* is not transmitted to retired agents on impact since their income merely relies on financial assets. On the other hand, labour income of the working age population is affected. An increase in the interest rate leads to workers increase their labour supply. At the same time, aggregate demand, and hence labour demand, falls leading to a further decline in the (real) wage. The demand of agents with relatively less asset holdings declines, in particular that of young workers. The effect of a decline in real wage is less detrimental for older people since the decline in real wage increases profits and hence dividend income of the asset holders. In Western EU countries, the accumulation of wealth is maximized at the retirement age.Therefore, as agents get older, we observe a higher decline in the labour supply (see Figure 6) and a lower decrease in consumption (see Figure 5). As life expectancy declines and/or wealth accumulates more, a labour supply decision of agents becomes more inelastic to changes in the wage level, i.e. the income effect of changes in the wage level has a smaller impact on the labour supply decision of the agents.

Figure 6
Responses of labour to 25 bps tightening monetary policy shock; Western EU countries

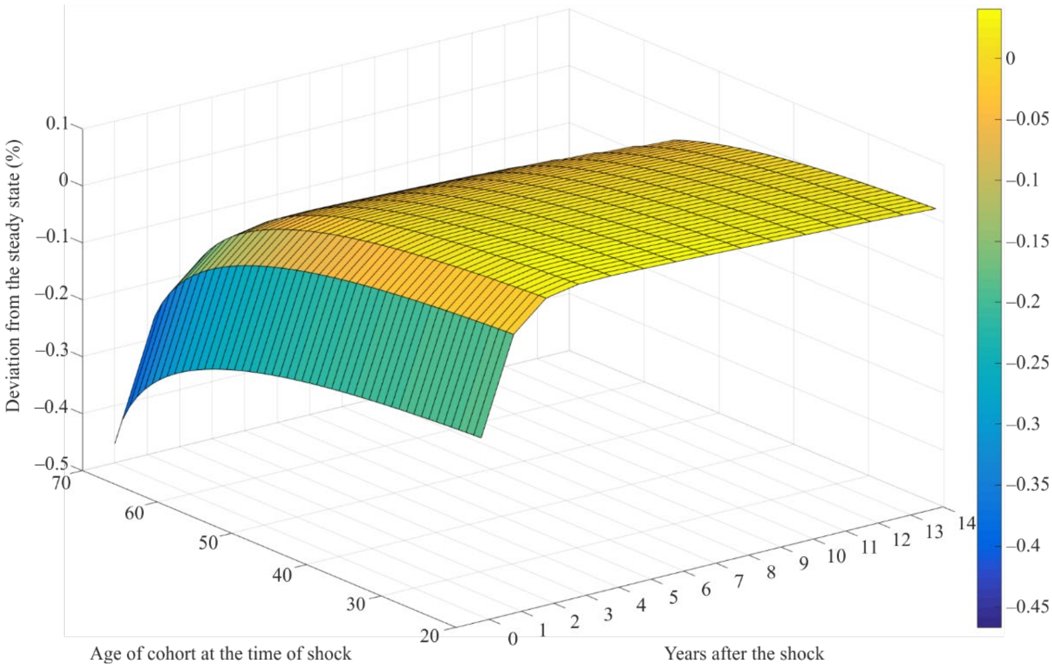


Figure 7 compares IRFs of consumption to a 25 bps tightening monetary policy shock under the assumption of 0.25% population and 2% productivity growth rate, which corresponds to the levels observed in Eastern EU countries, and 0% population and productivity growth rates representing Western EU countries for different age groups (20, 40, 50, 65, 70 and 80). The decline in consumption is higher for younger workers in former specification of the model. However, the consumption level of middle-aged workers decreases by the same amount in both cases. In Eastern EU countries, the productivity gap between cohorts leads to different levels of productivity-adjusted wage. Thus, due to the fact that the productivity level of middle-aged agents is lower than that of younger workers, the labour income of middle-aged agents decreases less. After an increase in the interest rate, young agents tend to supply more labour which results in further decline in wages compared to Western EU countries. The decline in the labour demand is stronger due to more pronounced decline in labour income, hence the aggregate demand declines more. The decrease in the wage level leads to an increase in dividend income. Thus, the fall in the consumption level in Eastern EU countries is partly offset by a higher return from asset holdings. Since wealth accumulation reaches the maximum around the age of 50–55 in Eastern EU countries, the slope of the initial responses of cohorts is steeper compared to Western EU countries.

Figure 8 illustrates the responses of the aggregate variables to a tightening monetary policy shock. Depending on the demographic composition and productivity level of society (and hence the distribution of the net wealth), the magnitude of the decrease in output varies. Since this is a demand-driven model and labour is the sole input in the production of final goods, the demand for labour decreases following the amplitude of decline in the level of output. The difference in productivity growth between cohorts realizes in more pronounced fall in real wages in Eastern EU countries and therefore also in the labour supply. In the New Keynesian model, the source of inflation is the marginal cost. Accordingly, in Eastern EU countries, we observe a stronger decline in inflation. This result is consistent with the empirical evidence provided in Feldkircher and Huber (2016), Fadejeva et al. (2017), Burriel and Galesi (2018) and Hajek and Horvath (2018), which show that the response of output and inflation to a monetary policy shock in Central and Eastern European countries could be larger compared to the one observed in other euro area countries.

Figure 7
Responses of consumption of cohorts to 25 bps tightening monetary policy shock

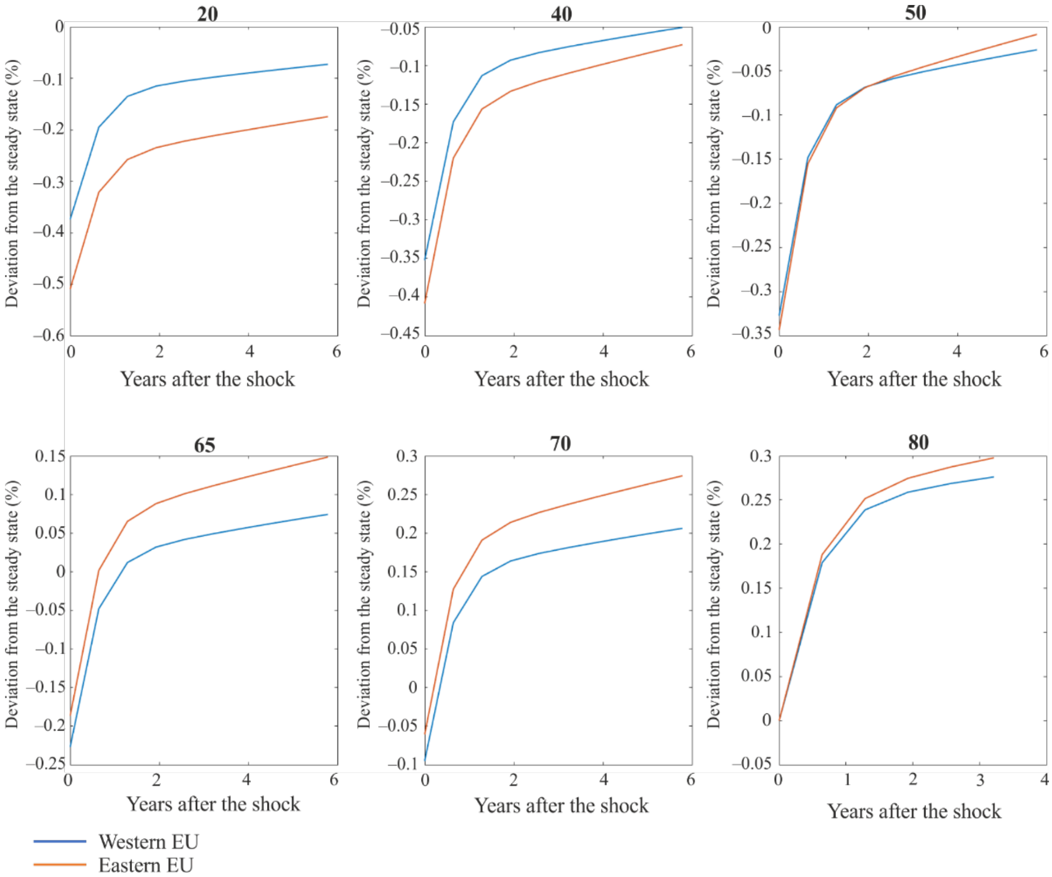
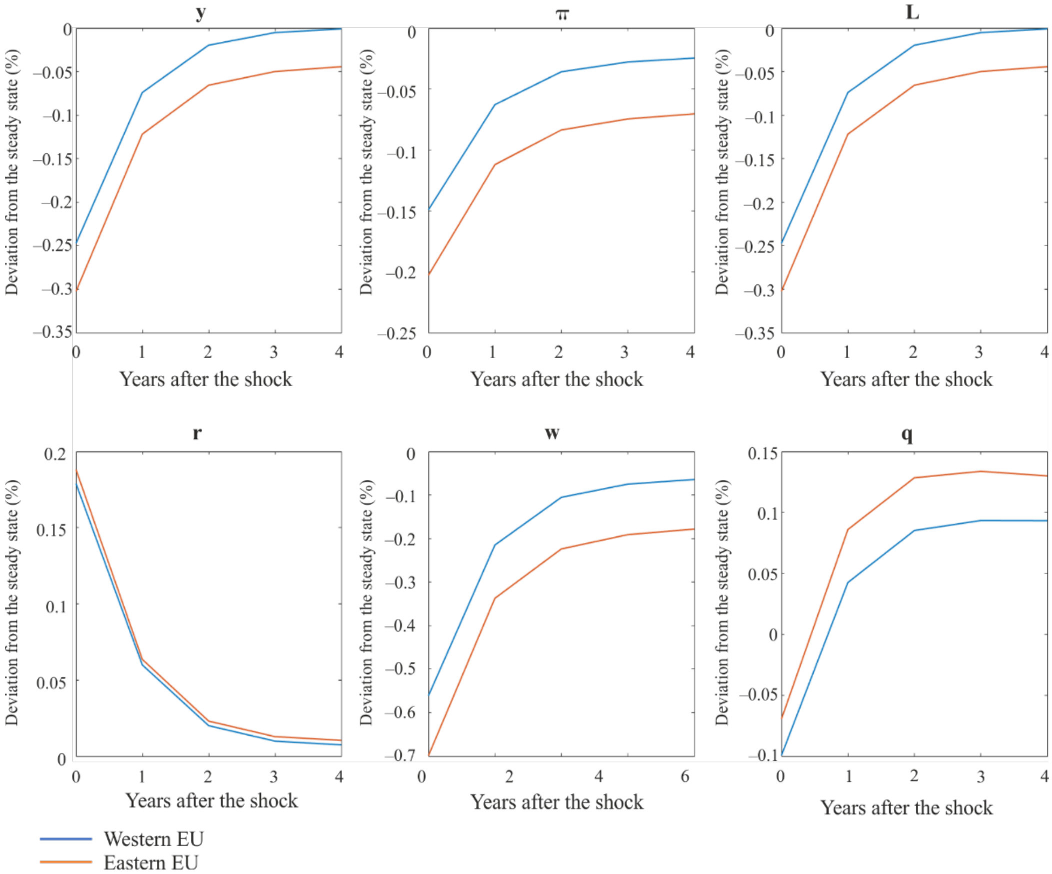


Figure 8
Responses to 25 bps tightening monetary policy shock

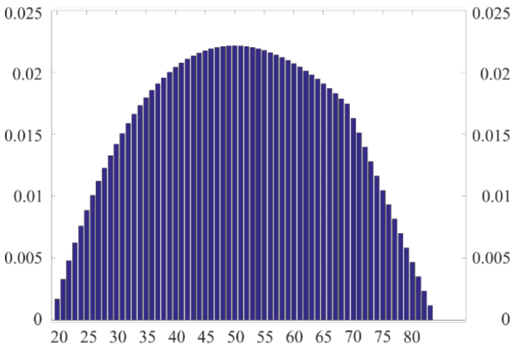


Moreover, in Western EU countries, the asset price falls more which can be explained by the abundance of financial assets held by the old agents in older economies compared to younger ones. Therefore, the response of asset prices to the same monetary policy shock is stronger in Western EU countries. During the retirement period, old agents sell their assets to dissave. In Eastern EU countries, wealth is accumulated before the retirement age due to generation heterogeneity, and people can start dissaving earlier. Thus, at retirement age, supply of financial assets is limited and a higher demand leads to a smaller decline in the price of an asset as compared to Western EU countries. Also, in Western EU countries, the supply of financial assets is higher due to a higher old-age dependency ratio therefore the price of an asset declines more, reaching the amount of an increase in the real interest rate.

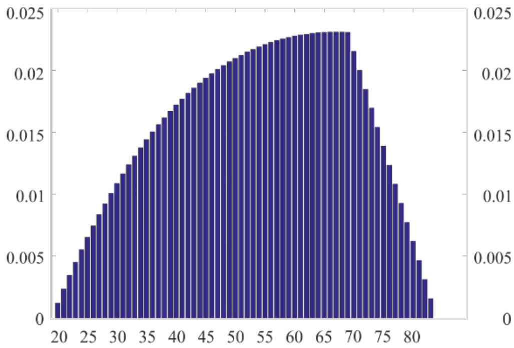
5.1 Sensitivity analysis

In addition to differences in productivity levels and demographic profiles, there are other possible factors affecting the shape of the net wealth distribution. The (inverse) IES, which measures the rate of adjustment in current consumption in response to a change in policy rate, is another candidate. Figure 9 displays the shapes of net wealth distribution for different IES. The left panel shows the distribution of net wealth by age in an economy with high IES preferences, $\sigma = 0.8$, and the right panel shows the case with a low IES, $\sigma = 1.5$.

Figure 9
Net wealth distribution by age



(a) σ = 0.8



(b) σ = 1.5

In the previous section, due to lack of precise value for this parameter, we assigned the same value to both groups of countries. However, narratively we observe cultural and social differences between Eastern and Western EU countries which could be explained by the IES. In this section, we compare impulse responses to a 25 bps tightening monetary policy shock, using the Eastern EU model specification with different σ levels: $\sigma = 0.8$, $\sigma = 1$ and $\sigma = 1.5$.[12] Figure 10 displays consumption impulse responses of different cohorts. Larger σ (low IES) implies stronger *wealth effect* and weaker *substitution effect* therefore the consumption level declines less for each age group. Moreover, as the IES gets larger, the impact of policy change on the consumption level (of cohorts) increases. The analysis can be formulated by the following equation (17). Under the assumption of higher IES, $\sigma = 0.8$, a higher growth rate of consumption is needed to balance out an increase in the interest rate.

$$\hat{\imath}_t - E_t\{\hat{\pi}_{t+1}\} = \sigma \Delta c_{t+1} \qquad (17).$$

[12] To save space, we report the results for Eastern EU countries only.

Figure 10

Response of consumption to 25 bps tightening monetary policy shock

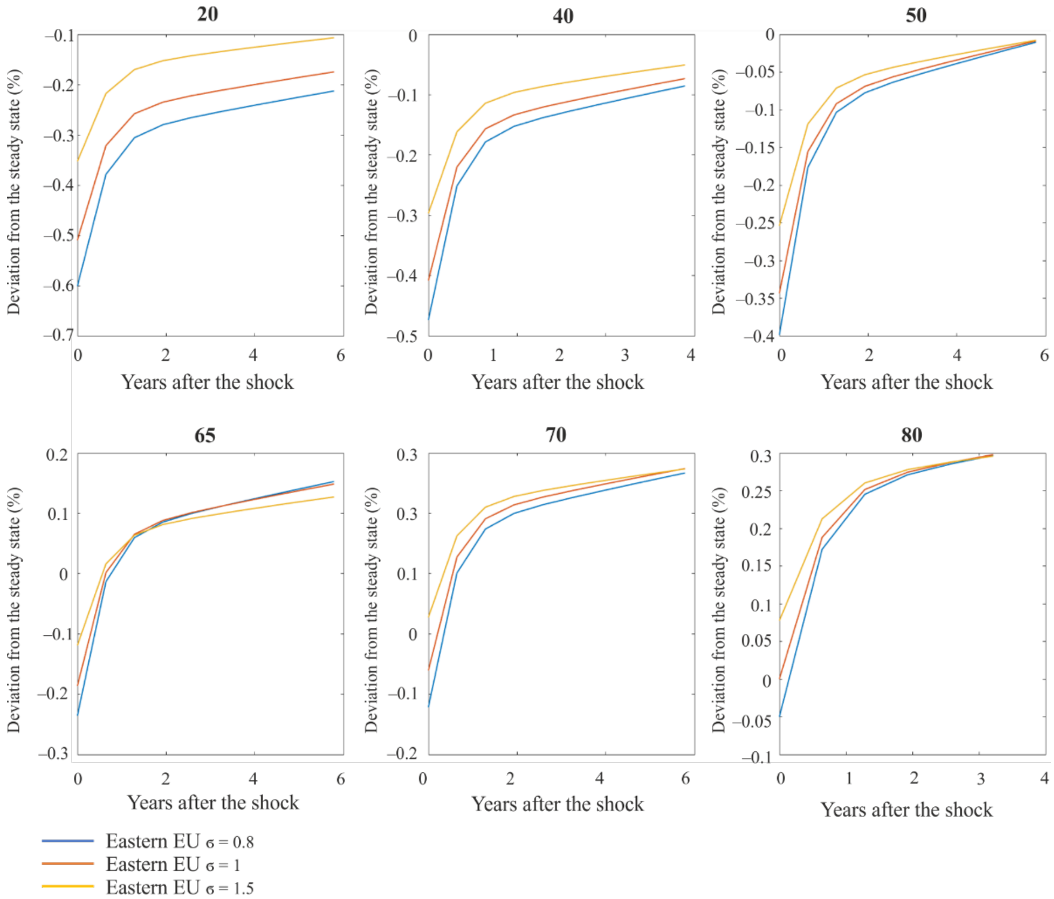


Figure 11
Response of labour to 25 bps tightening monetary policy shock

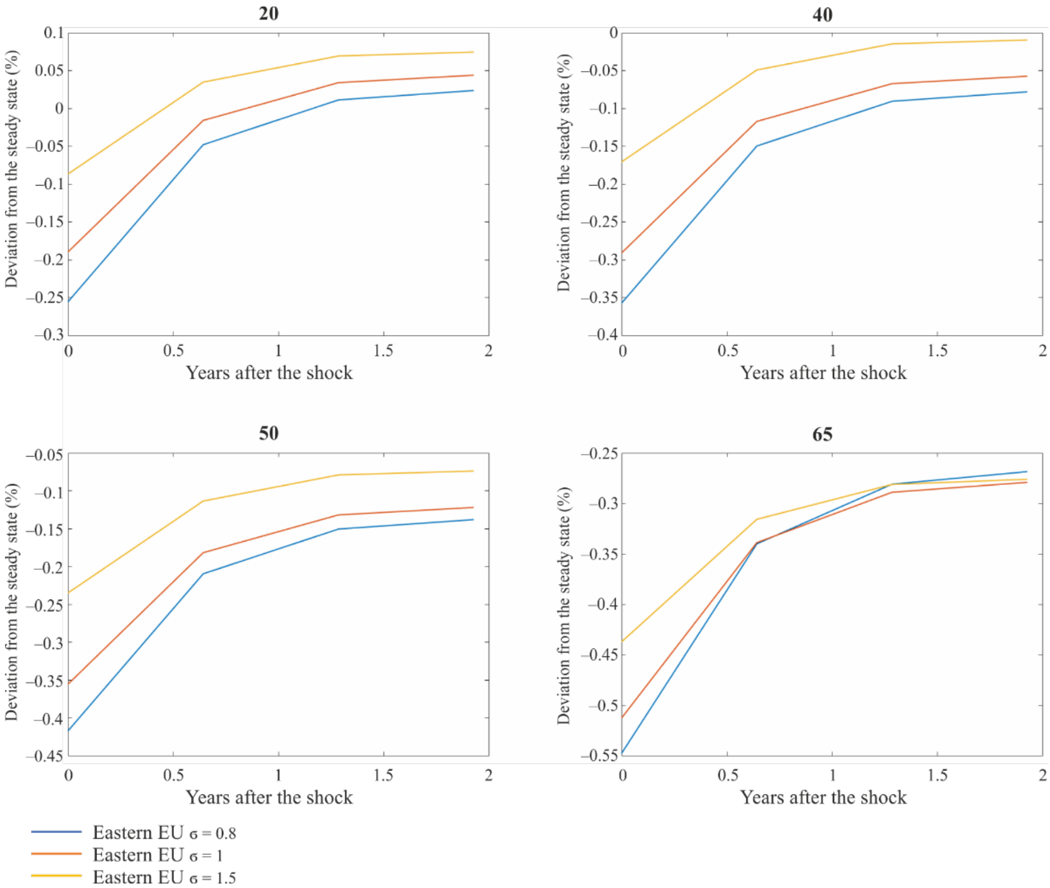
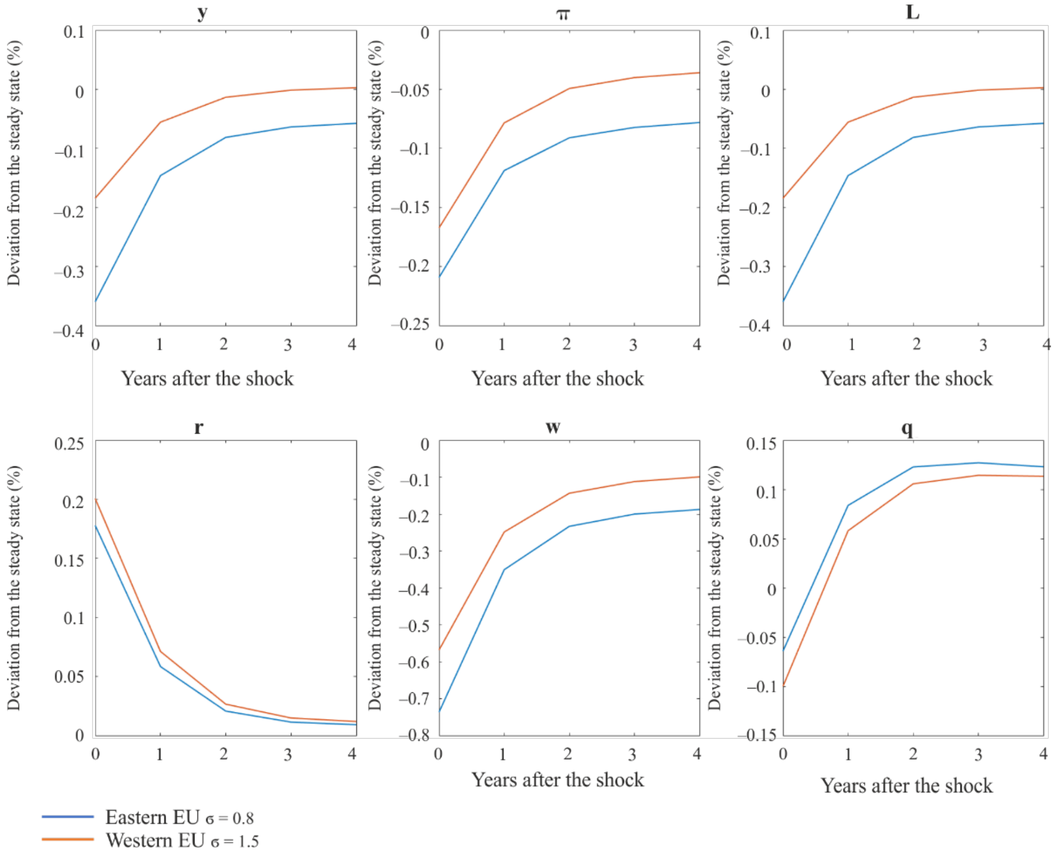


Figure 11 shows impulse responses of labour supply. In addition to the negative *income effect*, the substitution and wealth effect of interest rate change has a significant impact on the labour supply and hence the labour demand. Under the high IES model calibration, the *substitution effect* on consumption is stronger and the *wealth effect* is weaker. This means that agents prefer to postpone their consumption level more compared to the model with a lower IES value. The labour supply increases more with such preference, and the decline in the wage level and thus labour income is higher. Therefore, the reduction in the aggregate and individual labour demand is more pronounced.

Figure 12
Responses to 25 bps tightening monetary policy shock



Next, we provide impulse responses of key aggregate variables to a positive monetary policy shock under different assumptions on IES, generation productivity and population growth values. As shown in Figure 12, the blue line represents an economy with high productivity and population growth rates, and a high IES *(economy high)*; the red line represents an economy with low productivity and population growth rates, and a low IES value *(economy low)*. Tightening monetary policy rate has a stronger negative impact on all variables of the *economy high,* except asset prices. The stronger substitution effect of a high IES reinforces the impact of higher productivity differences among generations and higher population growth rate.

6. CONCLUSION

In the present paper, we seek to investigate whether differences in the wealth distribution by age in Eastern and Western EU countries has an impact on the effectiveness of monetary policy. In this context, first, we identify the source(s) of differences in the wealth distribution by age in these two groups of countries. Next, we develop a coherent theoretical model, which incorporates these features, to analyse the impact of monetary policy shocks.

Understanding the determinants of wealth distribution is a critical step. Until the end of the 20th century, Eastern EU countries were part of the centrally planned economic system. Due to the collapse of this system, we observe discrepancies in the productivity level between cohorts. Some implicit barriers hinder old workers from starting making use of the new resources bred by the new system which contribute to their productivity. Therefore, younger individuals earn higher labour income than their older colleagues. In the paper, we provide an evidence of a productivity gap between cohorts by showing the differences in wage profiles between countries by age. The productivity gap between cohorts accounts for the cross-country differences in the shape of age-net wealth distribution. This assumption suggests that young workers are more productive than the older ones at a given period of time. Hence, young agents, compared to the old ones, can accumulate more wealth. Moreover, we also show that the age structure of economies is a significant factor for the differences in wealth distribution in these two groups of countries.

Theoretically, we develop a modified New Keynesian model, which merges multiple period overlapping generations (OLG) and dynamic New Keynesian (DNK) frameworks. The demand side of the model assumes an OLG structure, which enables us to introduce a productivity gap between generations and demographic characteristics into the model economy. On the supply side, following the standard DNK setup of Galí (2015), the introduction of price rigidities to the model allows monetary policy to influence the interest rate and the real economy. The augmented framework is used to analyse the impact of wealth accumulation originated by demographics and the productivity gap among generations on the effectiveness of monetary policy in a coherent general equilibrium model.

We estimate the resulting model with HFCS data for Eastern and Western EU countries. We provided the evidence that the effect of monetary policy on output and inflation weakens as the net wealth distribution moves towards older ages, i.e. in Western EU countries. Furthermore, we also show that young agents in Western and Eastern EU countries respond differently to monetary policy shocks: monetary policy is more effective for younger agents of Eastern EU countries. Accordingly, our findings regarding the responses to monetary policy suggest that the net wealth distribution plays a key role in the effectiveness of monetary policy at both individual and aggregate levels. We also show that the natural rate of interest decreases monotonically not only as the OAD ratio increases, but also as the productivity gap among generations disappears. Overall, the findings of the paper suggest that a stronger reaction of Eastern EU countries to a monetary policy shock compared to Western EU countries can be partly attributed to the differences in the net wealth distribution by age.

APPENDIX A. SUPPLY SIDE OF THE ECONOMY AND DERIVATION OF THE PHILLIPS EQUATION

Consumption (final) goods producers:

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}}.$$

$\varepsilon > 1$: elasticity of substitution among the differentiated intermediate goods.

Y_t: final good.

$Y_t(i)$: intermediate good i.

The profit maximization problem of the final good producer is:

$$\max_{Y_t(i)} P_t Y_t - \int_0^1 P_t(i) Y_t(i) di$$

subject to

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}}$$

The first order condition(s):

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} Y_t.$$

The final (consumption) goods industry is perfectly competitive, thus

$$P_t Y_t = \int_0^1 P_t(i) Y_t(i) di,$$

$$P_t Y_t = \int_0^1 P_t(i) \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} Y_t di.$$

The aggregate price index is:

$$P_t = \left[\int_0^1 P_t(i)^{1-\varepsilon} di\right]^{\frac{1}{1-\varepsilon}}.$$

Intermediate goods producers:

The production function of the intermediate goods industry is:

$$Y_t(i) = Z_t N_t^1 + Z_{t-1} N_t^2 + \cdots + Z_{t-R-1} N_t^R$$

where N_t^k is the hours worked per worker of cohort k at time t. R is the retirement age. Z_t denotes the productivity level of the youngest cohort at time t and $Z_{t+1} = Z_t(1+g)$. The *productivity-adjusted* production function[13] is:

$$\mathcal{Y}_t(i) = \frac{Y_t(i)}{Z_t} = N_t^1(i) + \frac{N_t^2(i)}{(1+g)} + \cdots + \frac{N_t^R(i)}{(1+g)^{R-1}}.$$

[13] Variables are adjusted by the productivity level of the youngest cohort at a given time period.

Cost minimization

$$\min_{N_t^k(i)} W_t \left[N_t^1(i) + \frac{N_t^2(i)}{(1+g)} + \cdots + \frac{N_t^R(i)}{(1+g)^{R-1}} \right]$$

subject to

$$\mathcal{Y}_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} \mathcal{Y}_t,$$

$$\mathcal{Y}_t(i) = N_t^1(i) + \frac{N_t^2(i)}{(1+g)} + \cdots + \frac{N_t^R(i)}{(1+g)^{R-1}},$$

$$\mathcal{L} = -W_t[N_t^1(i) + \frac{N_t^2(i)}{(1+g)} + \cdots + \frac{N_t^R(i)}{(1+g)^{R-1}}] + \varphi_t[N_t^1(i) + \frac{N_t^2(i)}{(1+g)} + \cdots + \frac{N_t^R(i)}{(1+g)^{R-1}}$$

$$-\left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} \mathcal{Y}_t].$$

The first order conditions:

$$N_t^1(i) = -W_t + \varphi_t = 0,$$

$$N_t^2(i) = -\frac{W_t}{(1+g)} + \frac{\varphi_t}{(1+g)} = 0,$$

$$\vdots$$

$$N_t^R(i) = -\frac{W_t}{(1+g)^{R-1}} + \frac{\varphi_t}{(1+g)^{R-1}} = 0$$

where φ_t is the marginal cost at time t. $w_t = \frac{\varphi_t}{P_t} = \frac{W_t}{P_t}$ is the real marginal cost.

Profit maximization

$$\max_{P_t(i)} P_t(i) Y_t(i) - W_t L_t(i)$$

subject to

$$\mathcal{Y}_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} \mathcal{Y}_t,$$

$$W_t = \varphi_t$$

where

$$L_t(i) = \left[N_t^1(i) + \frac{N_t^2(i)}{(1+g)} + \cdots + \frac{N_t^R(i)}{(1+g)^{R-1}}\right] = Y_t(i).$$

The first order condition:

$$P_t(i) = \frac{\varepsilon}{1-\varepsilon} \varphi_t$$

where $\frac{\varepsilon}{1-\varepsilon}$ is the markup. At steady state $\varphi = \frac{\varepsilon}{1-\varepsilon} = 1/\mathcal{M}$.

Reoptimizing firms' profit, the maximization problem is:

$$\max_{P_t^*} E_t \sum_{k=0}^{\infty} \theta^k \tilde{\beta}^k \widetilde{\Lambda}_{t,t+k} (P_t^* Y_{t+k}(i) - \varphi_{t+k} Y_{t+k}(i))$$

subject to

$$\mathcal{Y}_{t+k}(i) = \left(\frac{P_t^*}{P_{t+k}}\right)^{-\varepsilon} \mathcal{Y}_{t+k}$$

where $\widetilde{\Lambda}_{t,t+k} = \left(\frac{c_{t+k}^{p+1}}{c_t^p}\right)^{-\sigma}$, where p denotes the cohort type.

The maximization problem becomes:

$$\max_{P_t^*} E_t \sum_{k=0}^{\infty} (\theta\tilde{\beta})^k \widetilde{\Lambda}_{t,t+k} \left(\frac{P_t^*}{P_{t+k}} \left(\frac{P_t^*}{P_{t+k}}\right)^{-\varepsilon} \mathcal{Y}_{t+k} - \frac{\varphi_{t+k}}{P_{t+k}} \left(\frac{P_t^*}{P_{t+k}}\right)^{-\varepsilon} \mathcal{Y}_{t+k}\right).$$

The first order condition:

$$E_t \sum_{k=0}^{\infty} (\theta\tilde{\beta})^k \widetilde{\Lambda}_{t,t+k} ((1-\varepsilon)(P_t^*)^{-\varepsilon} (P_{t+k})^{\varepsilon-1} Y_{t+k})$$
$$= E_t \sum_{k=0}^{\infty} (\theta\tilde{\beta})^k \widetilde{\Lambda}_{t,t+k} ((-\varepsilon)(P_t^*)^{-\varepsilon-1} \varphi_{t+k} (P_{t+k})^{\varepsilon-1} Y_{t+k}),$$

$$P_t^* = \frac{\varepsilon}{\varepsilon-1} \frac{E_t \sum_{k=0}^{\infty} (\theta\tilde{\beta})^k \widetilde{\Lambda}_{t,t+k} (\varphi_{t+k} \mathcal{Y}_{t+k} (P_{t+k})^{\varepsilon-1})}{E_t \sum_{k=0}^{\infty} (\theta\tilde{\beta})^k \widetilde{\Lambda}_{t,t+k} ((P_{t+k})^{\varepsilon-1} \mathcal{Y}_{t+k})},$$

$$P_t^* = \frac{\varepsilon}{\varepsilon-1} \frac{A_t}{B_t}$$

where

$$A_t = E_t \sum_{k=0}^{\infty} (\theta\tilde{\beta})^k \widetilde{\Lambda}_{t,t+k} (\varphi_{t+k} \mathcal{Y}_{t+k} (P_{t+k})^{\varepsilon-1}),$$

$$B_t = E_t \sum_{k=0}^{\infty} (\theta\tilde{\beta})^k \widetilde{\Lambda}_{t,t+k} ((P_{t+k})^{\varepsilon-1} \mathcal{Y}_{t+k}).$$

They can be written as:

$$A_t = \widetilde{\Lambda}_{t,t} \varphi_t \mathcal{Y}_t (P_t)^{\varepsilon-1} + \theta\tilde{\beta}\widetilde{\Lambda}_{t,t+1} E_t\{A_{t+1}\},$$

$$B_t = \widetilde{\Lambda}_{t,t} \mathcal{Y}_t (P_t)^{\varepsilon-1} + \theta\tilde{\beta}\widetilde{\Lambda}_{t,t+1} E_t\{B_{t+1}\}$$

where $\widetilde{\Lambda}_{t,t} = \left(\frac{c_t^{k+1}}{c_t^k}\right)^{-\sigma}$.

The aggregate price index is:

$$P_t = \left[\int_0^1 P_t(i)^{1-\varepsilon} di\right]^{\frac{1}{1-\varepsilon}},$$

$$P_t = \left[\int_0^1 [(1-\theta)(P_t^*(i))^{1-\varepsilon} + \theta(P_{t-1})^{1-\varepsilon}] di\right]^{\frac{1}{1-\varepsilon}}.$$

Divide both sides by P_{t-1}

$$\frac{P_t}{P_{t-1}} = \left[\int_0^1 [(1-\theta)(P_t^*(i))^{1-\varepsilon} + \theta(P_{t-1})^{1-\varepsilon}]di\right]^{\frac{1}{1-\varepsilon}} \frac{1}{P_{t-1}}$$

$$(1+\pi_t)^{1-\varepsilon} = (1-\theta)\left(\frac{P_t^*(i)}{P_{t-1}}\right)^{1-\varepsilon} + \theta \qquad (18)$$

$$\frac{P_t^*}{P_{t-1}} = \frac{\varepsilon}{\varepsilon-1}\frac{A_t}{P_{t-1}}\frac{1}{B_t}$$

$$\frac{A_t}{P_{t-1}} = \frac{1}{P_{t-1}}\left[\widetilde{\Lambda}_{t,t}\varphi_t \mathcal{Y}_t (P_t)^{\varepsilon-1} + \theta\tilde{\beta}\widetilde{\Lambda}_{t,t+1}E_t\{A_{t+1}\}\right]$$

$$\frac{A_t}{P_{t-1}} = \frac{\varphi_t}{P_t}\frac{P_t}{P_{t-1}}(P_t)^{\varepsilon-1}\mathcal{Y}_t\widetilde{\Lambda}_{t,t} + \theta\tilde{\beta}\widetilde{\Lambda}_{t,t+1}E_t\frac{A_{t+1}}{P_{t-1}}\frac{P_t}{P_t}$$

where $\frac{\varphi_t}{P_t} = mc_t^R$. Rearranging the above expression, we get

$$\tilde{A}_t = (1+\pi_t)[mc_t^R(P_t)^{\varepsilon-1}\mathcal{Y}_t\widetilde{\Lambda}_{t,t} + \theta\tilde{\beta}\widetilde{\Lambda}_{t,t+1}E_t\tilde{A}_{t+1},$$

$$\frac{P_t^*}{P_{t-1}} = \frac{\varepsilon}{\varepsilon-1}\frac{\tilde{A}_t}{B_t}.$$

Let $\tilde{a}_t = \frac{\tilde{A}_t}{(P_t)^{\varepsilon-1}}$ and $\tilde{b}_t = \frac{B_t}{(P_t)^{\varepsilon-1}}$.

$$\frac{P_t^*}{P_{t-1}} = \frac{\varepsilon}{\varepsilon-1}\frac{\tilde{a}_t}{\tilde{b}_t} \qquad (19),$$

$$\tilde{a}_t = \frac{\tilde{A}_t}{(P_t)^{\varepsilon-1}} = (1+\pi_t)[mc_t^R\mathcal{Y}_t\widetilde{\Lambda}_{t,t} + \theta\tilde{\beta}\widetilde{\Lambda}_{t,t+1}E_t(1+\pi_t)^{\varepsilon-1}\tilde{a}_{t+1}],$$

$$\tilde{b}_t = \frac{B_t}{(P_t)^{\varepsilon-1}} = \mathcal{Y}_t\widetilde{\Lambda}_{t,t} + \theta\tilde{\beta}\widetilde{\Lambda}_{t,t+1}E_t(1+\pi_t)^{\varepsilon-1}\tilde{b}_{t+1}.$$

At steady state

$$\tilde{a}^* = \frac{mc^*\mathcal{Y}^*\widetilde{\Lambda}^*}{1-\theta\tilde{\beta}\widetilde{\Lambda}^*},$$

$$\tilde{b}^* = \frac{\mathcal{Y}^*\widetilde{\Lambda}^*}{1-\theta\tilde{\beta}\widetilde{\Lambda}^*},$$

$$\frac{P^*}{P^*} = 1 = \frac{\varepsilon}{\varepsilon-1}\frac{\tilde{a}^*}{\tilde{b}^*} \quad \Rightarrow \quad mc^* = \frac{\varepsilon}{\varepsilon-1} = \frac{1}{\mathcal{M}}.$$

Log-linearize equation (19)

$$\hat{p}_t^* - \hat{p}_{t-1} = \hat{\tilde{a}}_t - \hat{\tilde{b}}_t \qquad (20),$$

$$\hat{\tilde{a}}_t = \pi_t + \frac{mc^*\mathcal{Y}^*\widetilde{\Lambda}^*}{a^*}[\widehat{mc}_t^R + \widehat{\widetilde{\Lambda}}_{t,t} + \hat{y}_t] + \theta\tilde{\beta}\widetilde{\Lambda}^*[\hat{\tilde{a}}_{t+1} + \widehat{\widetilde{\Lambda}}_{t,t+1} + (\varepsilon-1)\pi_{t+1}] \qquad (21),$$

$$\hat{\tilde{b}}_t = \frac{\widetilde{\Lambda}^*\mathcal{Y}^*}{\tilde{b}^*}[\hat{y}_t + \widehat{\widetilde{\Lambda}}_{t,t}] + \theta\tilde{\beta}\widetilde{\Lambda}^*[\hat{\tilde{b}}_{t+1} + \widehat{\widetilde{\Lambda}}_{t,t+1} + (\varepsilon-1)\pi_{t+1}] \qquad (22).$$

Plug equations (21) and (22) into (20):

$$\hat{p}_t^* - \hat{p}_{t-1} = \pi_t + (1 - \theta\tilde{\beta}\widetilde{\Lambda}^*)\widehat{mc}_t^R + \theta\tilde{\beta}\widetilde{\Lambda}^*(\hat{\tilde{a}}_{t+1} - \hat{\tilde{b}}_{t+1}) \quad (23).$$

Log-linearize equation (18)

$$\pi_t = (1 - \theta)[\hat{p}_t^* - \hat{p}_{t-1}] \quad (24).$$

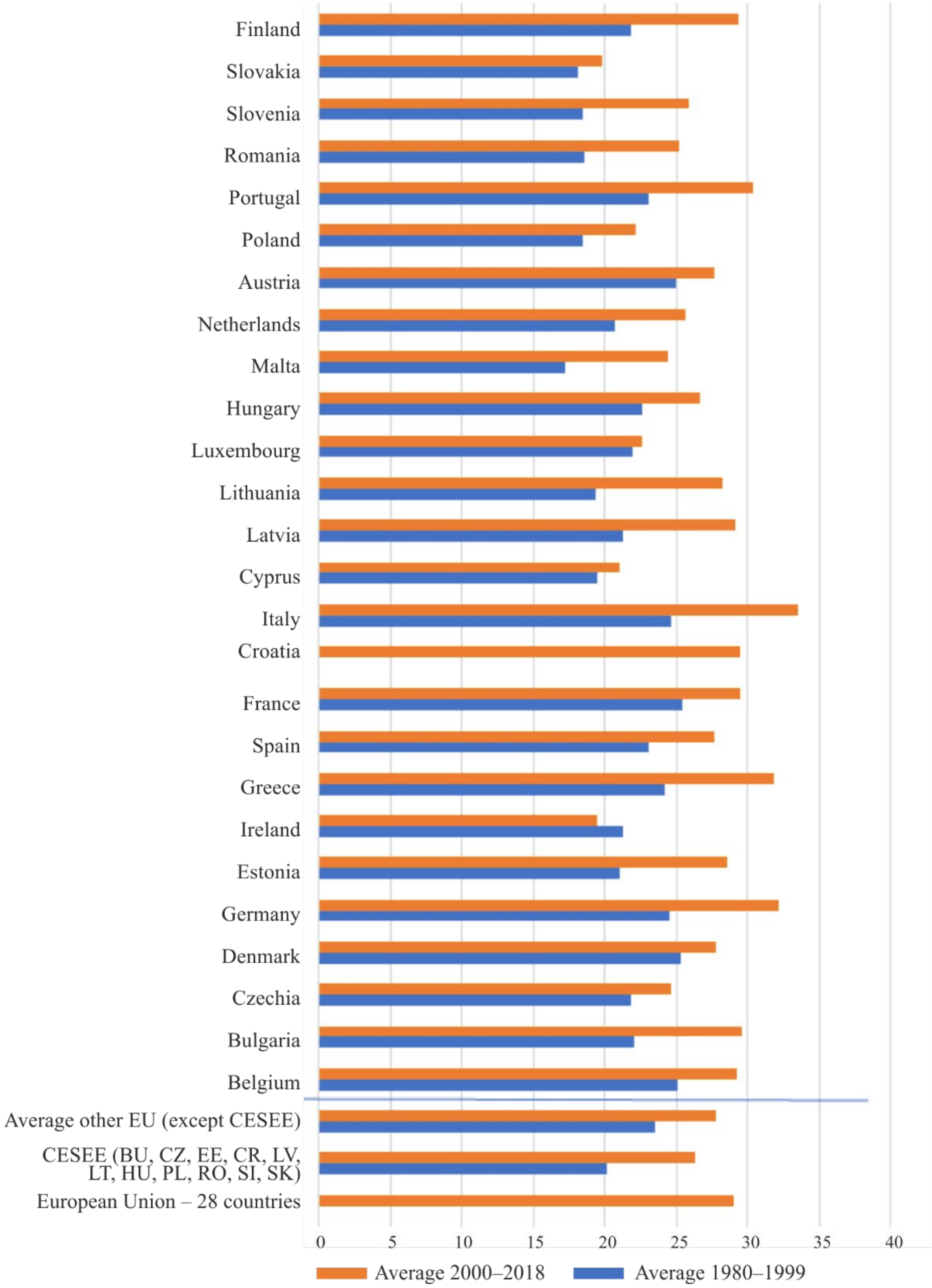
Plugging equation (23) into the above linearized expression, we get the forward-looking *Phillips equation*.

$$\pi_t = \tilde{\beta}\widetilde{\Lambda}^* E_t \pi_{t+1} + \frac{(1-\theta)(1-\theta\tilde{\beta}\widetilde{\Lambda}^*)}{\theta}\widehat{mc}_t^R \quad (25).$$

APPENDIX B. SOME DESCRIPTIVE STATISTICS

Old-age dependency ratio

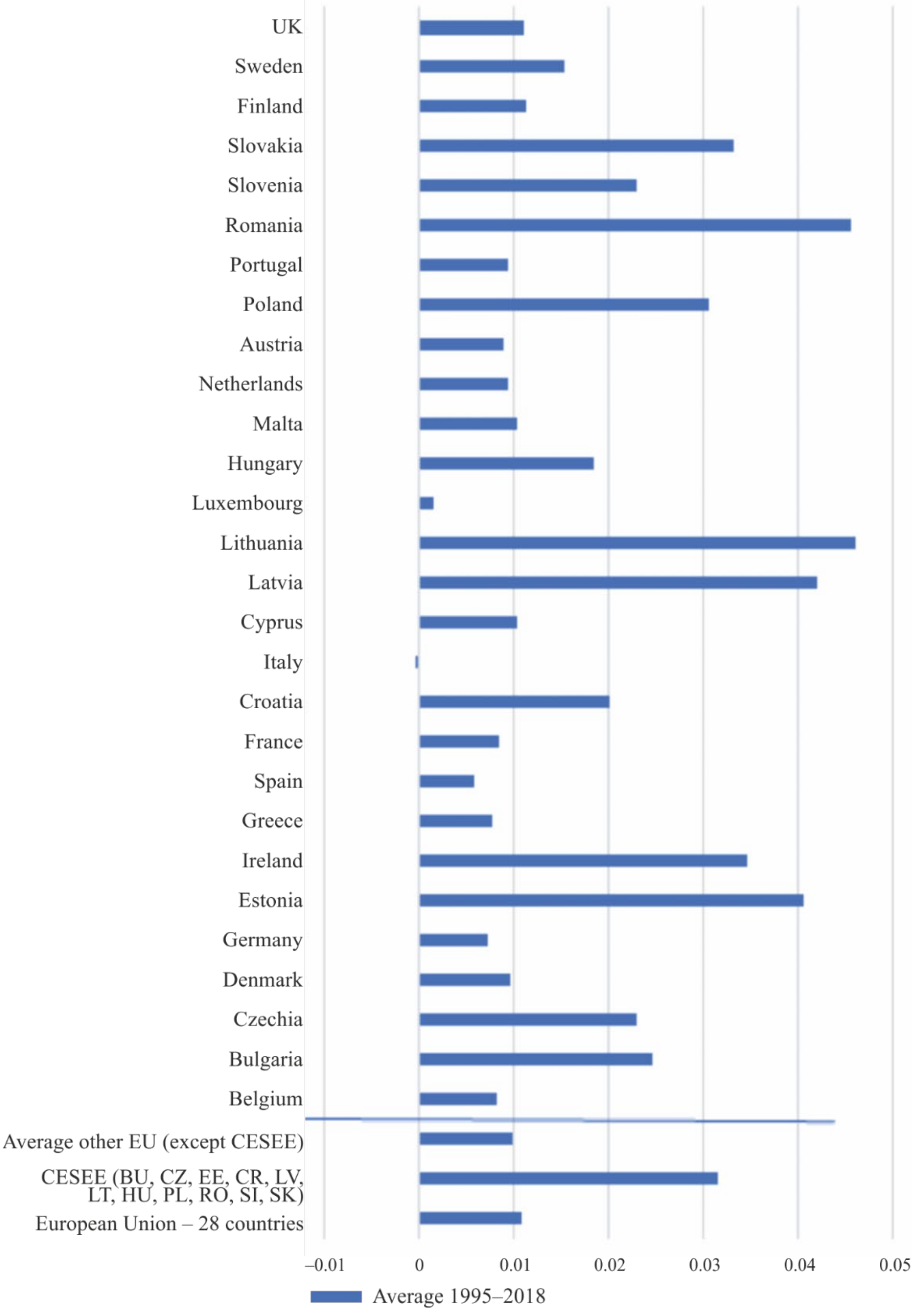
Figure B.1
Old-age dependency ratio
(% of population share 65+ to 20–64)



Source: Eurostat.

Productivity growth rate

Figure B.2
Annual real labour productivity growth per person
(1 = 100%)



Source: Eurostat.

APPENDIX C. BVAR DATA DESCRIPTION

Table C.1
BVAR data description

Variables	**Italy**	**Latvia**
Period (annual frequency)	1996–2018	2002–2018
Consumption	**Data source:** Central Bank of Italy, Historical Database of the Survey on Household Income and Wealth. Variables estimated per income earner in the household. **Available period:** 1996–2016 (bi-annual) **Variables used:** amount of expenditure on food (CONSAL) **Variable measurement:** annual growth rates **Age categories:** 20–39, 40–59, 60+ **Data adjustment:** since the survey is bi-annual, we estimate values of variables for years not covered by the survey, using simple averages. In 2017 and 2018, the growth rate of consumption for each group is equal to the household consumption growth from the National Accounts.	**Data source:** Central Statistical Bureau of Latvia, Household Budget Survey, Consumption expenditure by age of the main breadwinner average per household member per month, EUR **Available period:** 2002–2016 **Variables used:** consumption of food and non-alcoholic beverages **Variable measurement:** annual growth rates **Age categories:** 18–39, 40–59, 60+ **Data adjustment:** in 2017 and 2018, the growth rate of consumption for each group is equal to the household consumption growth from the National Accounts.
Inflation	**Data source:** Eurostat; **Variables used:** all-items HICP, annual average rate of change	
Interest rate	**Data sources:** IMF, Eurostat; **Variable used:** money market 3-month interest rate. For Italy, the euro area variable is used. For Latvia – the euro area estimate from 2014 is used.	

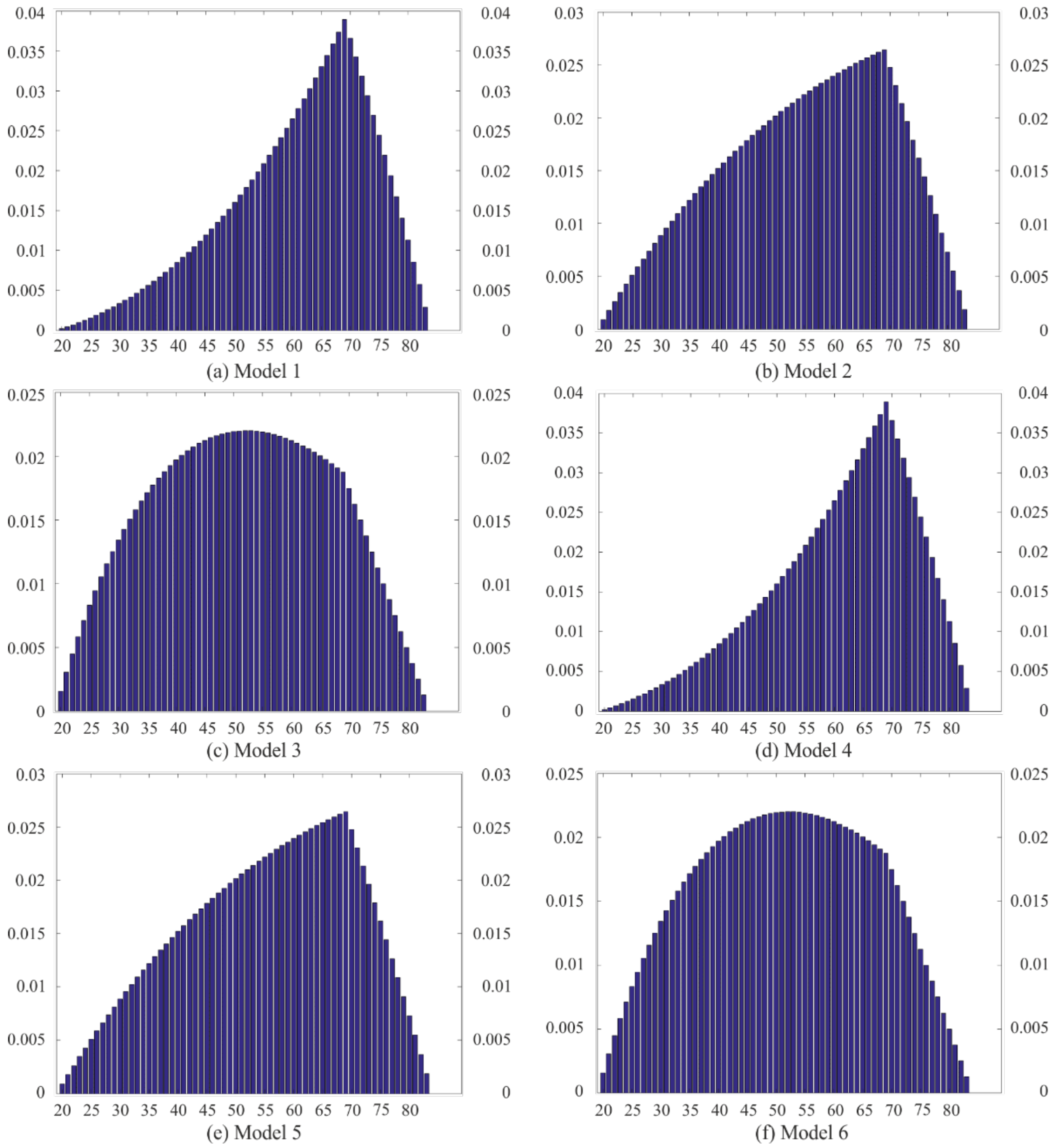
Figure C.2
Annual growth rate of food consumption by cohort
(%)

(a) Italy, food consumption
b) Latvia, food consumption

Note. See Figure 9 and Table C.1 for details on source and estimation.

APPENDIX D. WEALTH DISTRIBUTION

Figure D.1
Distribution of net wealth by cohort



APPENDIX E. RESPONSES OF CONSUMPTION AND LABOUR FOR EASTERN EU COUNTRIES

Figure E.1
Responses of consumption to 25 bps tightening monetary policy shock; Eastern EU countries

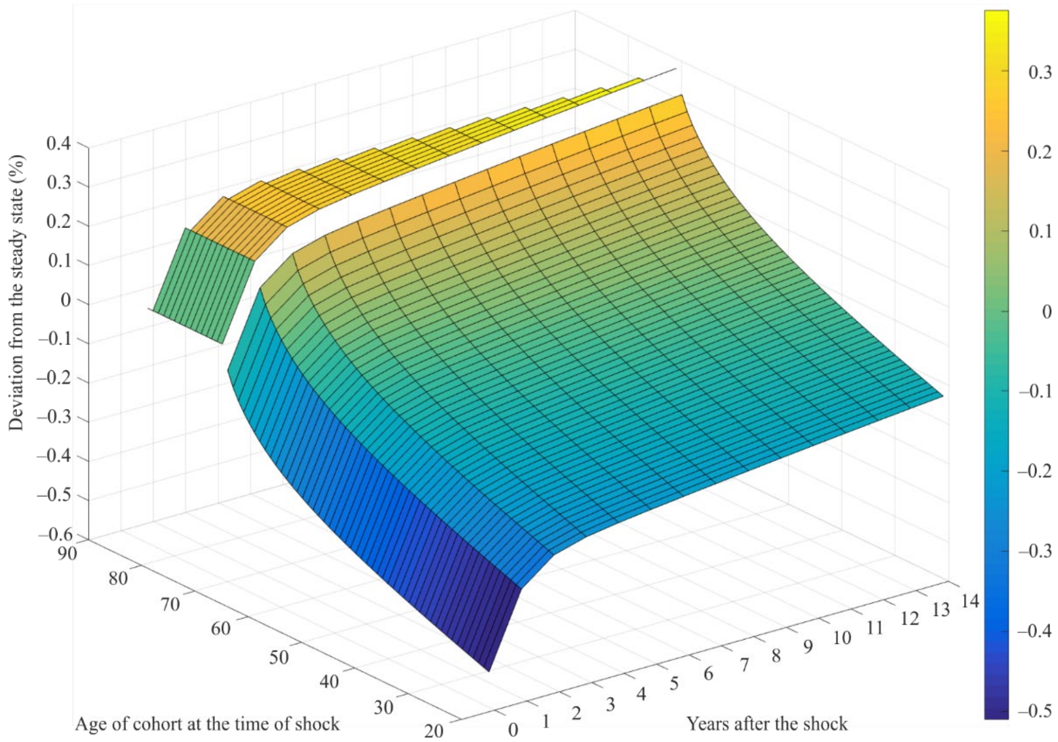
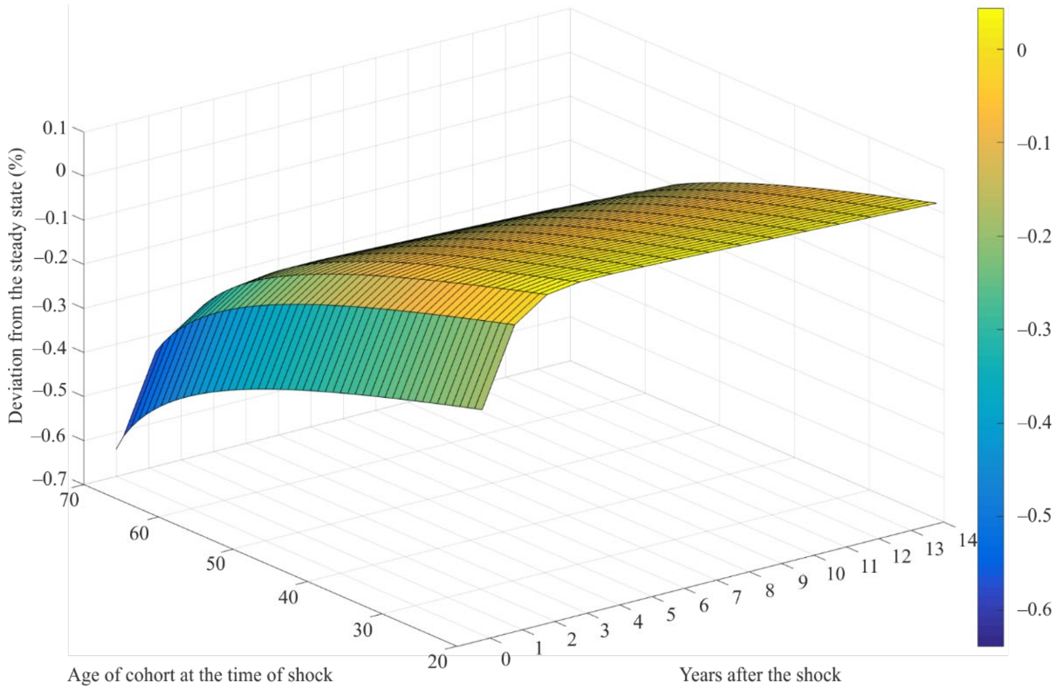


Figure E.2
Responses of labour to 25 bps tightening monetary policy shock; Eastern EU countries



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