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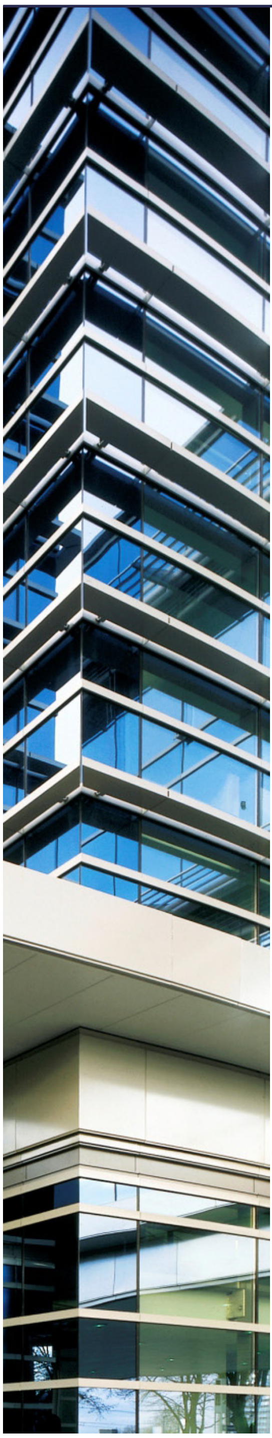

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One Who Hesitates Is Lost: Monetary Policy Under Model Uncertainty and Model Misspecification

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Abstract

This paper investigates how different parametrisation of the monetary policy reaction function and different mechanisms of expectations formation shape the macroeconomic outcomes in the Smets-Wouters type DSGE model. The initial macroeconomic conditions of the simulations correspond to the high inflation environment of early 2022. The simulation results show that under the hybrid expectations the terminal monetary policy rate is significantly higher than under the rational expectations for all Taylor rule parametrisations. Under the hybrid expectations, the inflation rate is much more persistent than under the rational expectations; three years is not enough to reach the inflation target of two per cent even for quite hawkish calibration of the Taylor rule. In the modelled economy, a relatively fast inflation stabilization for the hawkish Taylor rule has its own price in form of the cumulative output loss when compared with the dovish Taylor rule. Simulations are also performed for the case where the central bank misspecifies expectations formation mechanism in the DSGE model and follows an interest rate path implied by a false model. The results show that the hawkish reaction is preferable for both rightly and wrongly specified models.

Keyword: DSGE, Monetary policy, Expectations, High inflation, Loss function
JEL classification: C62, C63, D9, D58.

1 Introduction

Surging inflation in developed economies during 2021-2023 presented a significant challenge to central banks in their pursuit of price stability. In such circumstances, answers to questions how rapidly the inflation rate will return to the target level and what will be the price to pay in the form of cumulative output loss depend on the strength of the central bank policy response, i.e. how hawkish or dovish the central bank is when increasing the interest rate. At the same time, the economic outcomes of the monetary policy actions also depend on how the economic agents react to various shocks and the actions of the central bank, which, in turn, is conditional on the expectations formation mechanism of the economic agents.

Dynamic stochastic general equilibrium (DSGE) models allow understanding of the transmission mechanism of monetary policy actions and obtaining quantitative estimations of the macroeconomic impact of monetary policy decisions. Therefore, DSGE models are widely used for policy simulations. However, the use of models is not without caveats, particularly considering the uncertainty of economic models and their forecasts, which can present a problem in the decision-making process of monetary policy. Among uncertainties related to DSGE models is the specification of how agents form their expectations, and the potential variation of parameters over time. Since different assumptions can significantly alter predicted macroeconomic outcomes, understanding these sensitivities is critical for policymakers when using such models as input.

Studies investigating the impact of uncertainty surrounding the model coefficients on optimal monetary policy date back to Brainard (1967), who showed that when the parameter that links the policy instrument to the target variable is uncertain, the policy should be less aggressive. However, Brainard's results were obtained under a rather simple set-up with only one parameter being uncertain. The subsequent literature considers the uncertainty surrounding multiple parameters and finds the opposite to be true; see Soderstrom (2002), Kimura and Kurozumi (2007), and Cateau and Murchison (2010), among others. They argue that central banks should respond more aggressively when they are uncertain about the model parameters.

Another type of uncertainty central banks have to deal with is the uncertainty regarding the mechanism of expectations formation. Although the rational expectations (RE) assumption is still widely used in DSGE modeling, the empirical literature finds evidence of deviation from it. Specifically, Landier et.al. (2017) in an experimental study find that rational expectations are rejected by the data for most participants in the experiment. Moreover, expectations are influenced by previous forecasts and tend to exaggerate the impact of the most recent shocks. Pfajfar and Zoakeli (2014) find that expectations are heterogeneous, with some subjects behaving in line with RE, while others adhering to adaptive learning methods. Using the Survey of Professional Forecasters, Coibion and Gorodnichenko (2015) find the underreaction of consensus forecast relative to the predictions of the RE model. Broer and Kohlhas (2018) also analyze survey data and find that forecasters revise their forecasts more than what is implied by the RE model. This has direct implications for practical policy making: whereas in the world of rational expectations temporary inflation shocks can be "looked through" as they do not affect agents' medium-term inflation expectations in a meaningful way, such temporary shocks can turn out to be more persistent and lead to a potentially more significant deviations from central banks' targets if agents are less forward-looking and pay more attention to the current inflation rate when making their consumption and saving decisions.

Finally, as has been rather obvious from the rather poor recent track record in terms of inflation forecasting across almost all major central banks, we have to acknowledge in the analysis the possibility that the models used by most central banks are not always perfect. In order to address the problem of model uncertainty, two approaches are often used in the literature: the first, the Bayesian approach (Cogley et.al. (2011)) that weighs each possibility of model specification by its prior probability; the second, the robust control or minimax (Hansen and Sargent (2007)), where a policy maker aims to minimise the outcome of the worst-case scenario.

This paper investigates how assumptions regarding these uncertainties and different parameterisations of the monetary policy reaction function shape macroeconomic outcomes in the standard Smets-Wouters type DSGE model (Smets and Wouters (2003) and Smets and Wouters (2007)). We **first** compare the optimal policy response given different forms of expectations formation, namely backward-looking expectations with elements of learning, and RE.[1] **Second**, in a similar way as in the robust control theory, we assume that the central bank may use wrong models of expectations formation to define a path of future interest rate, and then see what this means for inflation and output. We also consider cases where the central bank is "learning" about the true state of the economy together with other economic agents, and after observing the actual incoming data and comparing it to previous model predictions, it is ready and willing to adjust its views; particularly in case the central bank realises that it has used an incorrect model, it can switch to the correct model after some time. **Third**, we analyse different forms of potential non-linearities in the conduct of monetary policy. In the high inflation environment, a central bank may hike the interest rate using the conventional linear Taylor rule until the inflation is reduced considerably or commit to keep the interest rate at a somewhat lower level, but for a longer time. The latter circumstance is often called the "higher-for-longer" approach and can be modeled by the Taylor rule with a threshold.

In general, our results show that the degree to which a central bank should be aggressive in the face of high inflation depends on the weight it assigns to output loss in its objective function. These results indicate that a central bank with a strict price stability mandate, i.e. output loss weight in the objective function is low, should be more hawkish in its conduct of monetary policy. This holds true irrespective of whether the model used by the central bank is the correct one or not. Our findings also reveal that, compared to the policy implied by a linear Taylor rule, the higher-for-longer policy provides a noticeable reduction in cumulative output loss with a very small increase in cumulative inflation.

Among the works devoted to the monetary policy analysis during the recent surge of inflation in industrial countries the closest to ours are series of papers by the IMF (Alvarez et.al. (2023), Dizioli (2023) and WEO (2023)). They develop a DSGE model with a mix of forward- and backward-looking agents. They highlight the trade-off of bringing inflation down quickly and avoiding a significant loss in output. Their results also reveal that with the larger share of backward-looking agents in the economy, inflation prolongs, monetary policy weakens, and the output costs of monetary tightening rise. Our simulations also support these findings.

Our paper is different from the literature in several ways. First, most of the papers focus on impulse response functions for demand, supply, and monetary policy shocks, while we consider the forecasted dynamics of the economy with high initial inflation and filtered values for initial value of all state variables. This approach allows us to analyse the development of the macro-variables in a more comprehensive way. Second, the loss function in our welfare analysis is the deviation of inflation from the target and cumulative output loss after three years, but the IMF papers focus on the usual quadratic loss function. The loss function introduced in our study may reflect the preferences of policy makers more appropriately. Third, we also provide the welfare analysis for two types of uncertainties regarding: (1) expectations formation mechanism; and (2) whether the right or wrong model is used by the central bank.

The paper is organised as follows. Section 2 describes the DSGE model used in simulations. Section 3 discusses expectations formation mechanisms in DSGE models. Section 4 presents simulation results for models with rational and hybrid expectations and various parametrisations of the Taylor rule; Section 5 considers simulations for the case where the central bank uses a wrong model. Sections 6 and 7 address the implications of the simulation results in terms of the loss function and the expected loss function, respectively. Section 8 discusses the consequences of the delay in the monetary policy response if a central bank uses a wrong model. Section 9 compares

[1] Our modeling framework may be regarded as that of model coefficients uncertainty, because for the zero coefficients related to backward looking expectations and learning, the model corresponds to RE.

non-linear policy responses, such as the higher-for-longer, to linear Taylor rule policies. Section 10 provides the conclusions of the study.

2 Modelling setup

This paper uses the Smets-Wouters (Smets and Wouters (2003) and Smets and Wouters (2007)) model. In this model, there is a continuum of households, who supply household-specific labour in monopolistic competition and set wages. There is a continuum of intermediate good firms, who supply intermediate goods in monopolistic competition and set prices. Final goods use intermediate goods and are produced in perfect competition. To provide a reasonable fit of the model to the data for euro area, a number of real and nominal frictions are introduced: staggered prices and wages, price and wage indexation, Kimball aggregation, investment adjustment cost, and habit formation in consumption. The following shocks affect the economy: total factor productivity, investment-specific technology, household preference, exogeneous spending, price mark-up, wage mark-up, monetary policy, and risk premium.

The monetary policy reaction function is defined by the Taylor rule:

$$R_t = \rho R_{t-1} + (1-\rho)\left(r^* + \pi^* + \psi_p\left(\pi_t^{(4)} - \pi^*\right) + \psi_y y_t^{gap}\right) \tag{1}$$

where R_t is the annual nominal interest rate in time t; r^* is the annual real natural rate of interest fixed at 0.5% (inverse discount factor minus one), $\pi^* = 2.0\%$ is the annual steady state inflation, $\pi_t^{(4)} = \sum_{i=1}^{4} \pi_{t-i+1}$ is the annual inflation. The output gap, y_t^{gap}, is defined as the difference between the actual output and the potential one that corresponds to the equilibrium of flexible prices. The interest smoothing parameter is calibrated as $\rho = 0.85$, which corresponds to the value used in Cecion et.al. (2021). The benchmark specification assumes an output gap coefficient ψ_y = 1 and an inflation coefficient ψ_p= 1.5. As counterfactual, a more hawkish reaction function is considered with ψ_p = 2, 4 and 7.

The model parameters are obtained by applying the Bayesian estimation to the linearised model, and using data spanning 1999Q1-2014Q2. This the sample covers the period from the inception of euro until the the euro area policy interest rate breached the zero lower bound (ZLB). The sample is chosen to avoid the issue of non-linearity implied by the ZLB, thus allowing the use of the linearised model for estimation. The euro area macroeconomic time series include the same observable variables as in Smets and Wouters (2003): real GDP, real consumption, real investment, the GDP deflator, the real wages, employment, and the nominal interest rate.

Most of the calibrated parameters are set to the same values as in Smets and Wouters (2007). Specifically, the depreciation rate is 0.0025 per quarter, the gross mark-up on wages is 1.5. Share of government spending in output is 0.18. The curvature of the Kimball aggregator for wages and prices is set at 10. Exceptions include the steady-state inflation, which is set at 2%, and the discount rate, which equals 0.125 implying a discount factor of 0.99875. Additionally, we calibrate the parameters of the Taylor rule as described above. The remaining parameters are estimated. Information regarding the prior distribution, as well as the estimated mean, standard deviation, and posterior density 90% intervals for the parameters is provided in Appendix A.

3 Expectations formation in DSGE models

The standard assumption in DSGE modeling posits that agents have RE implying that they have complete knowledge of the underlying structure of the economy and that they make optimal decisions. Moreover, they are able to solve and estimate a DSGE model and, based on the obtained solution, make their forecasts on the true probabilistic expectations of the model's variables.

However, as mentioned in the Introduction, empirical studies show that the RE hypothesis is rejected for most individuals and that expectations are influenced by previous forecasts and tend to exaggerate the impact of recent shocks. Moreover, the assumption of RE in DSGE models can produce peculiar outcomes such as the forward guidance puzzle, i.e. an overly effective imp-act on the economy resulting from an announced future interest rate changes by the central bank (Del Negro et.al. (2012)). To address these issues, an HE formation mechanism has been proposed (see, Gertler (2017), and Walsh (2019), which incorporates past observations and model-based forecasts in agents' expectations. This approach produces better out-of-sample forecast properties than the RE assumption. This paper examines a particular specification of the HE mechanism proposed in Cecion et.al. (2021):

$$\mathbb{E}_t x_{t+1} = \alpha \mathbb{E}_t^{RE} x_{t+1} + (1-\alpha)\mathbb{E}_t^{AE} x_{t+1} \tag{2}$$

$$\mathbb{E}_t^{AE} x_{t+1} = \delta \mathbb{E}_{t-1}^{AE} x_t + (1-\delta) x_t \tag{3}$$

where x_t is a forward-looking variable of interest, $\mathbb{E}_t^{RE}$ is an expectation operator under RE, $\mathbb{E}_t^{AE}$ is an expectations operator under autoregressive expectations, α is a fraction of agents who understand the model and forecast the variable x_t according to the RE solution. The fraction $(1-\alpha)$ uses a learning scheme with an autoregressive component. In addition, those agents also update their beliefs according to the actual realisation of the variables of interest, $(1-\delta)x_t$. If $\alpha = 1$, expectations are fully rational, and if $\alpha = 0$, expectations are fully backward-looking. A degree of backward-looking behaviour of 0.8 is chosen for both parameters (α and δ) as in Cecion et.al. (2021). The mixture of rational and adaptive expectations is applied to prices. The application of a mixture of expectations to wages does not significantly change the results.

4 Results

4.1 Model simulations under rational and hybrid expectations and different parametrisation of the Taylor rule.

The conventional DSGE modeling involves assuming that an economy is in its steady state before being impacted by a shock. Impulse response functions are used to illustrate how the economy adjusts back to its steady state following the shock. In fact, these functions are forecasts of the deviation of endogenous variables from their steady state, under the initial conditions set as one standard deviation for a shock and as the steady state for all other endogenous variables. Various forms of impulse response functions, even for the same variable, but to different shocks, illustrate the significant influence of initial economic conditions on the forecasted path of macrovariables.

Instead of simulating impulse response functions for different shocks at the steady state of the economy, we focus on dynamics of macrovariables with initial conditions obtained by employing the Kalman filter, as implied by the DSGE model, to euro area data up to 2022Q2. Specifically, the initial value for inflation is set at 8.6% and for output growth at 1.1%. With these initial conditions established, we proceed to compute the inflation, output growth and nominal interest rates using the DSGE model. We consider three types of monetary policy reaction functions with different inflation coefficients: a. benchmark ψ_p=1.5; b. hawkish ψ_p=4; and super hawkish ψ_p=7. The last coefficient is chosen to guarantee inflation being close to the target after 8 quarters under the HE models.

Table 1 shows the results of the simulations. Columns 3 and 4 indicate the inflation rate at the end of 8 and 12 quarters, respectively. Under RE, only the hawkish reaction functions, ψ_p=4, can reach the inflation target after 8 quarters, with inflation being 2.01%. After 12, quarters the target of 2% is reached in nearly all forms of reaction function. Under HE, the terminal rates are higher than for RE by 1.7 times for the benchmark and 1.3 for the aggressive reaction functions, ψ_p=4. After 8 quarters, both the benchmark and the hawkish reaction functions do not provide inflation close to the target, 4.91% and 2.685%, respectively. After 12 months, the aggressive reaction function, ψ_p=4, entails the inflation rate relatively close to the target, namely 2.18%; whereas the benchmark reaction function provides the inflation rate of 3.72%. The cumulative output loss is about two times higher for HE than for RE for each reaction function.

Table 1: Model implied outcome with different parametrisation of the Taylor rule and different forms of expectations. Notes: the rows represent scenarios with different values of the inflation coefficient in the Taylor rule $\psi_p = 1.5, 2, 4$, and 7. HE features backward-looking expectations and elements of learning.

Inflation coefficients	**Terminal monetary policy rate**	**Inflation after 8 quarters**	**Inflation after 12 quarters**	**Cumulative output loss**
Rational expectations				
ψ_p=1.5	3.41	2.44	2.15	1.38
ψ_p= 2	3.73	2.29	2.1	1.72
ψ_p=4	5.51	2.01	2.02	2.61
ψ_p=7	8.47	1.85	2.01	3.42
Hybrid expectations				
ψ_p=1.5	5.76	4.91	3.72	2.90
ψ_p=2	5.68	4.03	3.04	3.40
ψ_p=4	7.19	2.68	2.18	5.00
ψ_p=7	10.41	2.03	1.92	6.35

Under the super-hawkish reaction function, inflation reaches the target for HE, nut at the costs of double output loss compared with the benchmark case for both RE and HE. The terminal rate for ψ_p=7 is also much higher than for the benchmark and hawkish reaction functions – 8.47% for RE and 10.41% for HE. Under the super-hawkish reaction function, inflation undershoots after two quarters for RE and after three quarters for HE. The intuition behind these results is as follows. The presence of agents with backward-looking expectations in the HE model implies a higher persistence of inflation as deviations from the central bank's target are not expected to automatically disappear but are instead gradually morphing into higher inflation expectations, thus potentially creating a self-enforcing inflationary loop. This results in a significantly slower inflation decay than under RE. Consequently, the central bank has to raise the interest rate to a higher degree under the HE. As a result, the terminal monetary policy rate is higher. Forward-looking agents in the HE model internalise this information and reduce their consumption and investment to a greater extent than in the RE model. For lower degree of backward behavior, $\alpha = \delta = 0.5$, the results obtained are somewhere between RE and HE with $\alpha = \delta = 0.8$ (see Appendix B).

4.2 Model simulations with uncertainty about the central bank assumptions

In Section 4.1, we examine scenarios where the central bank employs the correct model. Here, we assume that the central bank instead uses an incorrect model to determine the future path of the nominal interest rate. Once the central bank sets this path, it adheres to it, disregarding the nominal rate suggested by the Taylor rule. At the same time, the central bank communicates this projected interest rate path to rational economic agents, who incorporate this information into their decision-making, assuming they know the correct model. This setup parallels Type I ambiguity as defined by Hansen and Sargent (2012), where private agents know the correct probability model, while a central authority – represented here by the central bank – does not. We add another dimension assuming that the central bank can be either dovish (ψ_p=1.5) or hawkish (ψ_p=4). For the technical details of the scenarios implementation, see Appendix E

Figure 1 shows model implied inflation paths under different central banks reactions in the case when the HE model is correct, i.e. there is a significant share of agents in the economy which form inflation expectations based on the current inflation rates. The blue line represents an inflation path for the dovish central bank which has mistakenly assumed most agents in the economy are fully rational, has therefore followed the interest rate path implied by the RE model, and hence reacts (ex-post) too weakly to inflation deviation from the target. As a result, after 12 quarters inflation remains notably above the target of 2% and is higher than the inflation rate implied if the dovish central bank would have used the correct HE model and acted more forcefully (gray line vs blue line).

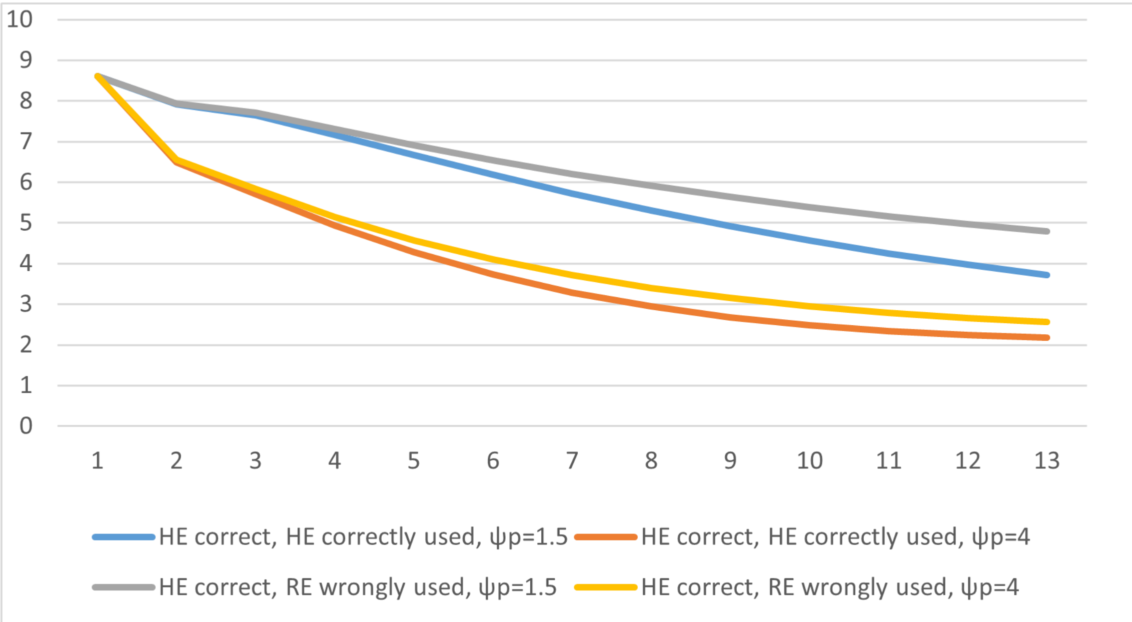

Figure 1: Model implied inflation rate path in the HE world and different monetary policy rules. Notes: ψ_p = 1.5 represents a dovish central bank; and ψ_p = 4 represents a hawkish central bank. Correct/false indicates if the central bank has used the correct (in this case HE) model when determining the interest rate path.

The hawkish central is able to bring down inflation to 2.18% in the third year if the central bank uses the correct HE model (orange line) and 2.56% if it uses the false RE model (yellow line). So, using the false RE model does not allow for reaching the target within a period of three years, although the difference in inflation between correct and false models used is much smaller than in the case of dovish central bank. In other words, being more hawkish brings lower penalty, in form of inflation overshot, when using the wrong model in an HE environment. So, for an inflation targeting central bank, uncertainty whether the inflation expectations are following an RE or HE model would imply the bias towards a more "hawkish" policy stance, as it would allow to minimise potential policy mistakes (in terms of larger and longer lasting inflation deviations from the target).

If we assume that an RE model is correct (Figure 2), but the central bank uses the HE model's interest rate path, the inflation rate comes down relatively fast, reaching 2% after 5 quarters under hawkish reaction function (yellow line) and after 7 quarters under dovish reaction function (orange line). In both cases, inflation undershoots the target level of 2% afterwards, but for the hawkish reaction function, inflation bottoms out earlier, tending then towards the steady state of 2%, whereas for dovish reaction function inflation bottoms out later. As a result, after 12 quarters inflation under the hawkish regime is a bit higher than under the dovish one, 1.79% vs 1.50%.

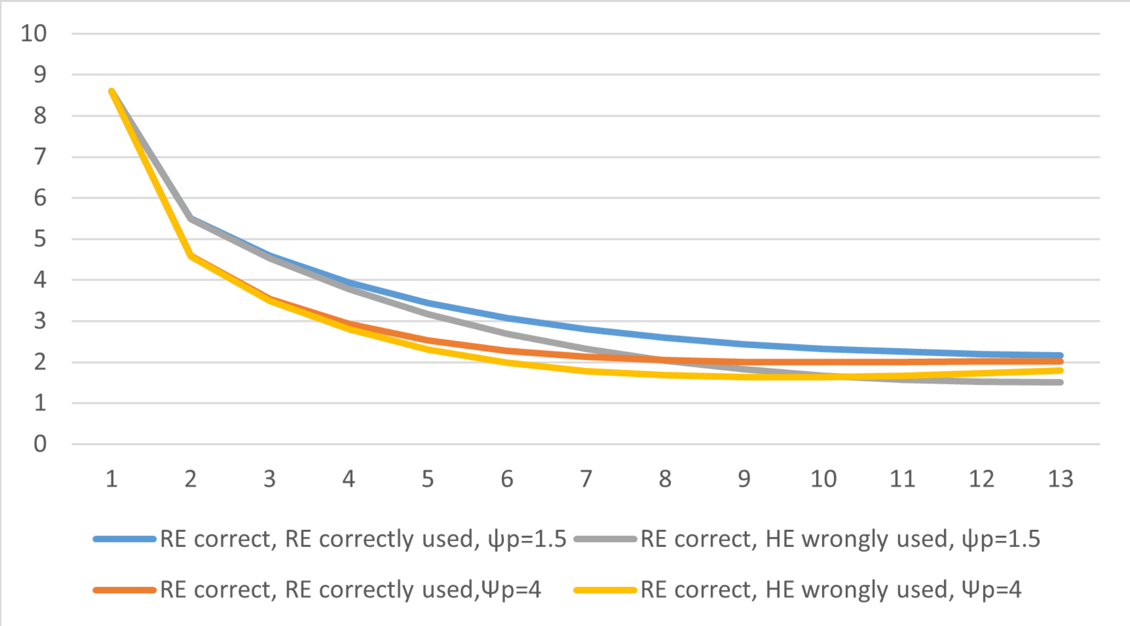

Figure 2: Model implied inflation rate path in the RE world and different monetary policy rules. Notes: ψ_p = 1.5 represents a dovish central bank; and ψ_p = 4 represents a hawkish central bank. Correct/false indicates if the central bank has used the correct (in this case RE) model when determining the interest rate path.

Overall, however, being in the RE world is much more beneficial for the central bank, as potential policy mistakes produce much smaller inflation deviations from the target. So again, if in the RE world it does not matter much which type of models central banks are using, while it does matter in the HE world where potential policy mistakes of “dovish” biases are more significant than those of the “hawkish” biases, then from the risk management perspective (if the central bank wants to avoid the worst-case scenario) it makes sense to assume, as the default option, that we might be living in the HE world, and the right approach for the central bank would be to be more “hawkish”, at least initially, until a clearer picture emerges. Initially, the interest rate is considerably higher under hawkish reaction function than under the dovish one (Figure 3). The terminal rate is higher by more than 1 pp. As a result, the inflation rate decreases faster for aggressive reaction function.

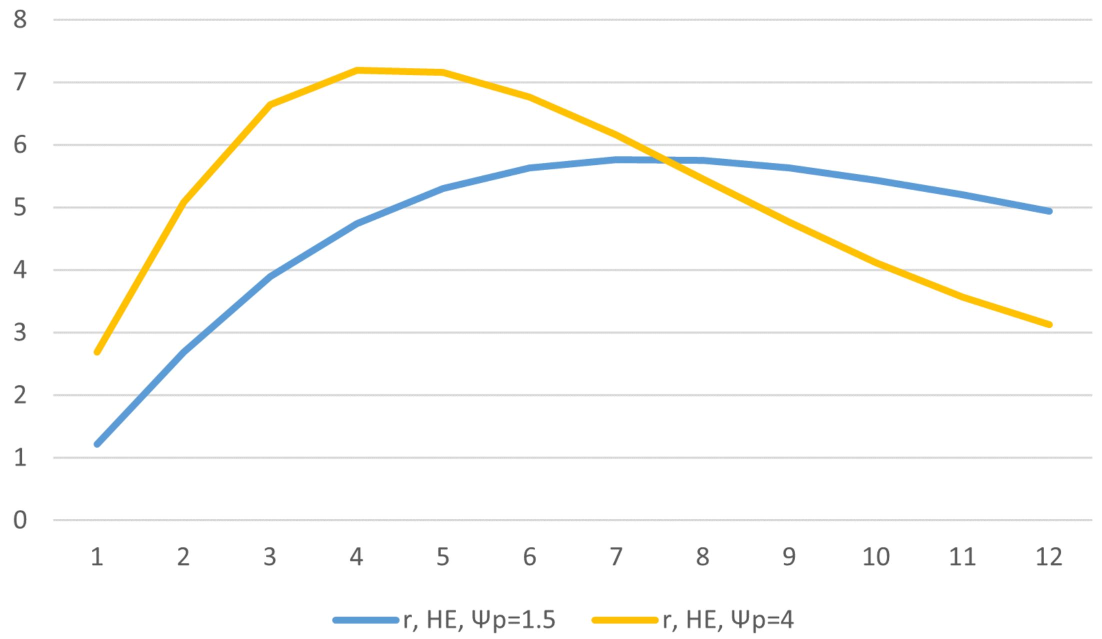

Figure 3: Loss functions depending on the weights assigned to the cumulative output loss when the central bank uses correct models. Notes: ψ_p =1.5 (dovish reaction); and =4 (hawkish reaction). HE features backward-looking expectations and elements of learning, RE stands for rational expectations.

Table 2 summarises key variables from all scenarios analysed in this section with an additional case of super-hawkish monetary policy defined in Section 4, ψ_p=7. We are particularly interested in the penalty of using the false model with an incorrect assumption of expectations formation, i.e. setting the nominal rates based on the HE model in a RE world and vice versa. It follows that falsely using the HE model when the real world is better represented by the RE model leads to undershooting of inflation and noticeable loss in cumulative output. This holds true for both the hawkish and the dovish reaction. In turn, falsely using the RE model instead of the HE model results in notably higher inflation and not reaching the inflation target after 12 quarters even for the hawkish monetary policy rule. In this scenario only the super hawkish reaction function, ψ_p=7, allows, to some extent, to reach the inflation target after 12 quarters, again, at the costs of loss in cumulative output.

Table 2: Model implied inflation after 8 and 12 quarters and cumulative output loss under different model specifications and monetary policy rules. Notes: rows represent scenarios assuming different values of the inflation coefficient in the Taylor rule, ψ_p =1.5 (dovish reaction); and ψ_p =4 (hawkish reaction) and ψ_p=7 (super hawkish reaction) . HE features backward-looking expectations and elements of learning. Upper and lower panes identify which model describes the economy correctly, i.e. the RE world or HE world. The RE model and HE model shows which model is used by the central bank to set the nominal interest rate.

Inflation coefficients	**Inflation after 8 quarters**	**Inflation after 12 quarters**	**Cumulative output loss**
Rational expectations world			
ψ_p**=1.5**			
RE model	2.44	2.15	1.38
HE model	1.82	1.50	2.88
ψ_p**=4**			
RE model	2.01	2.02	2.61
HE model	1.63	1.77	3.37
ψ_p**=7**			
RE model	1.85	2.01	3.42
HE model	1.52	1.78	4.36
Hybrid expectations world			
ψ_p**=1.5**			
HE model	4.91	3.72	2.89
RE model	5.64	4.79	1.20
ψ_p**=4**			
HE model	2.68	2.18	5.00
RE model	3.06	2.53	4.21
ψ_p**=7**			
HE model	2.03	1.92	6.35
RE model	2.39	2.13	5.48

4.3 Loss function considerations

To compare different monetary policy reaction functions under different (mis)specifications of expectations formation, we introduce the loss function as a weighted sum of absolute value for the deviation of inflation from the target (2%) at the 12^{th} quarter and cumulative output loss over 12 quarters:

$$Loss = |\pi_{12} - 2| + w_y * y_{loss}, \tag{4}$$

where π_{12} is inflation after 12 quarters, y_{loss} is the cumulative output loss, w_y is the weight assigned to the cumulative output loss in the loss function. Figure 4 shows the computed losses as a function of the weight w_y attached to output loss for correctly specified models, i.e. in those scenarios in which the central bank faces no uncertainty about how expectations are formed. Under the RE, the dovish policy is better than the hawkish policy except for the functions which assign very small weight to the cumulative output loss. In those cases, the hawkish policy is a little better. Under the HE, the hawkish policy is much better than the dovish for all reasonable scenarios.

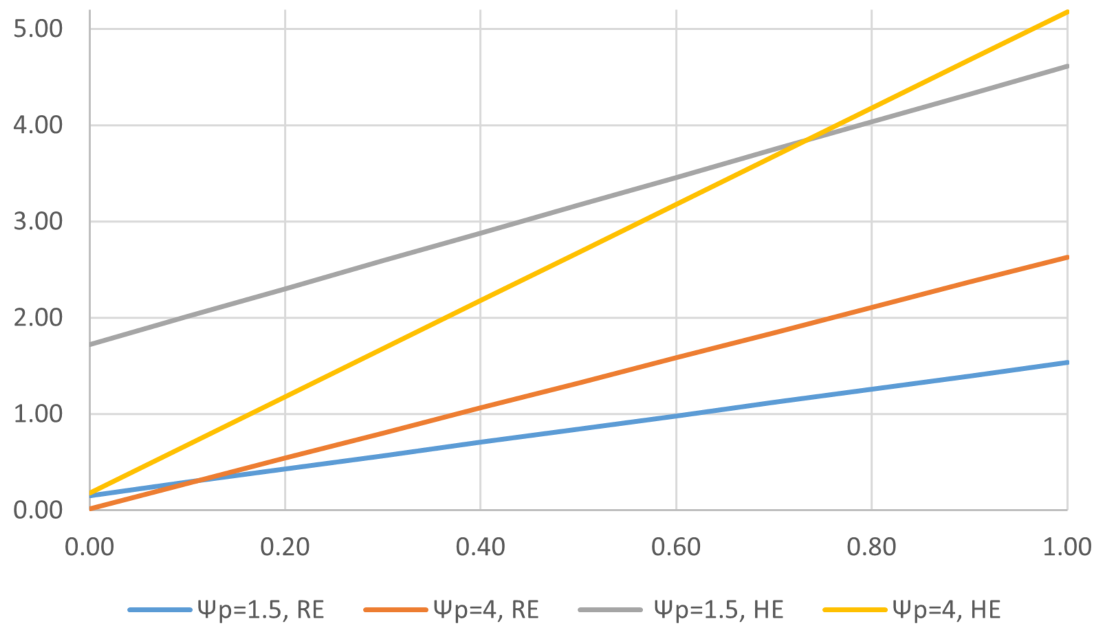

Figure 4: Loss functions depending on the weights assigned to the cumulative output loss when the central banks use false models. Notes: ψ_p =1.5 (dovish reaction); and ψ_p =4 (hawkish reaction). HE features backward-looking expectations and elements of learning, RE stands for rational expectations.

Figure 5 plots the loss function under the assumption that central bank uses in the model with the incorrect expectations formation mechanism. Incorrectly using the HE model instead of the RE model, the hawkish central bank generates lower loss than the dovish one if the weight of the cumulative output loss is less than roughly 0.5; however, the difference between the two is small (the blue line corresponds to the dovish CB, and the orange line corresponds to the hawkish CB). In the world of HE, but incorrectly using the RE model, the hawkish CB generates much lower loss than the dovish one in most of the cases, while only for quite large weights (>0.75), the dovish reaction is preferable. Overall, in both cases, for models specified rightly or wrongly, the hawkish reaction is the better choice if the weight assigned to the cumulative output loss is small.

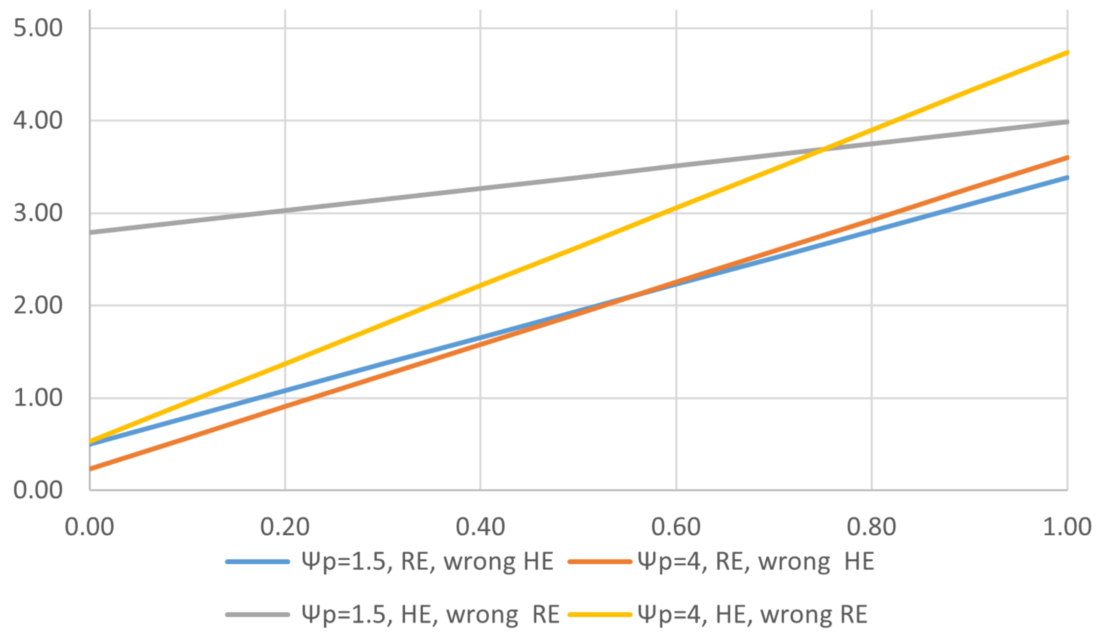

Figure 5: Loss functions depending on the weights assigned to the cumulative output loss when the central banks use false models. Notes: ψ_p =1.5 (dovish reaction); and ψ_p =4 (hawkish reaction). HE features backward-looking expectations and elements of learning, RE stands for rational expectations.

4.4 Policy making under double uncertainty

In the modeling framework described above, the central bank deals with the two types of uncertainties: (a) which expectations formation mechanism is correct, RE or HE; (b) whether the central bank uses the right or wrong model. Thus, there are four outcomes of these uncertainties: (1) the RE world and the central bank uses the RE model; (2) the RE world and the central bank uses the HE model; (3) the HE world and the central bank uses the RE model; (4) the HE world and the central bank uses the HE model. Under the assumption of equal prior probability for each outcome, the expectations of loss function may be written as:

$$E(loss) = \omega_1 L_{RE}^{RE} + \omega_2 L_{HE}^{RE} + \omega_3 L_{RE}^{HE} + \omega_4 L_{HE,}^{HE}, \quad (5)$$

where ω is the probability of a given outcome (we assume that $\omega_1 = \omega_2 = \omega_3 = \omega_4$=0.25). Figure 6 plots the expected loss as a function of the weight w_y for the hawkish (orange line) and dovish (blue line) central banks. If weight for output loss is less than 0.6, the expected loss of the hawkish central bank is less than that of a dovish one. An aggressive monetary policy is much better than the moderate one for small weights assigned to the output loss as supposed to be the case for central banks with the main mandate of price stability.

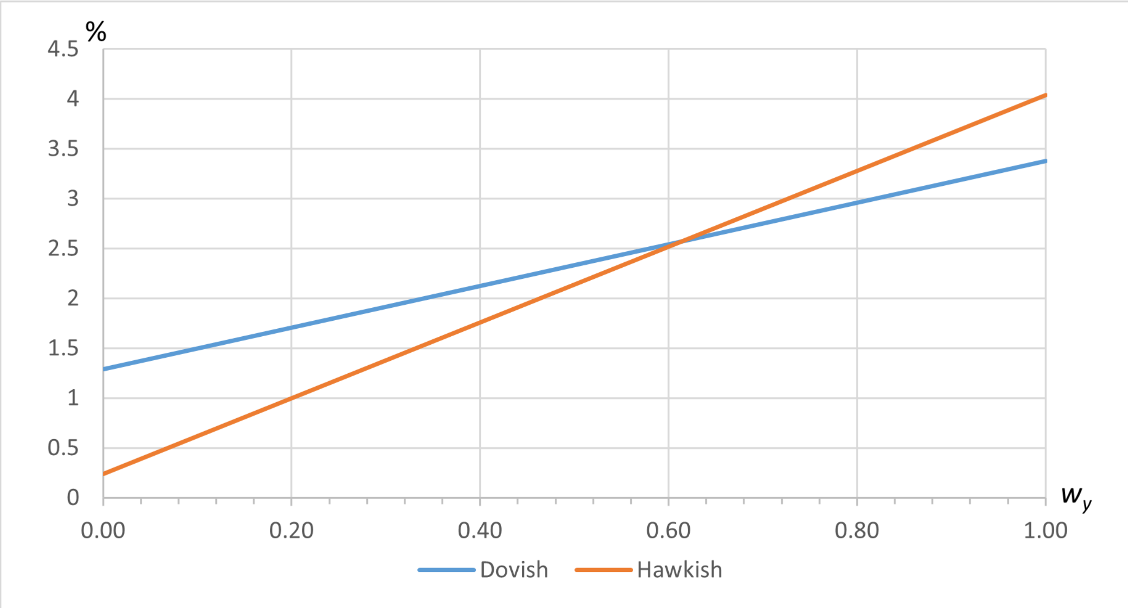

Figure 6: The expected loss functions by the weight attached to the cumulative output loss for the hawkish and dovish central banks. Notes: ψ_p =1.5 (dovish reaction); and ψ_p =4 (hawkish reaction).

4.5 Switching the model and reaction function

Assume that the dovish central bank uses the wrong RE model to determine the future path of the nominal interest rate. However, after 4 or 8 quarters it realises that inflation is too high and starts to be hawkish and use the correct HE model instead. This represents a case when central bank changes the model used and adjusts its reaction function. Next, we consider the following five cases: **Case 1**, as a benchmark, in which the dovish central bank uses the wrong RE model over the whole horizon; **Case 2**, as a second benchmark, the hawkish central bank uses the correct HE model from the start; **Case 3**, the central bank starts out with an incorrect RE model and dovish reaction, but switches to the HE model and being hawkish after 4 quarters. However, it has to be very hawkish now, or else it risks serious de-anchoring of inflation expectations, and permanent deviation of inflation from the target; and **Case 4**, the dovish central bank had used the wrong RE model for 8 quarters, given that the reaction from the central bank comes with a larger lag, the size of the adjustment of policy stance has to be even larger than in the previous case (ψ_p=10). For the technical details of the scenarios implementation, see Appendix E

The modelling results are summarised in Table 3. In the case of switching to the aggressive reaction function and correct model after 4 quarters (Case 3), the central bank reaches inflation of 2.26% after 12 quarters. Interestingly, after 16 quarters the inflation rate is 2.07%, which is even less than in the case of following the right HE model and aggressive monetary policy from the beginning (Case 2). This can be explained by noticing that for the former case, the much higher interest rate after 4 quarters pushes down inflation more strongly than for the latter case. The cumulative output loss for Case 3 is by 1.3 pp higher than for Case 2.

In the case of switching to the aggressive reaction function and the correct model after 8 quarters (Case 4), the inflation rate after 12 quarters is relatively high (2.86%) and reaches 2.18% after 16 quarters. The cumulative output loss for Case 4 is less than for Case 3 and even less than for Case 2, since hawkish policy response kicks in rather late in the horizon of the scenario. Finally, in Case 5 where the dovish central bank had used the wrong RE model for 8 quarters, then tries to bring inflation close to target by the 12^{th} quarter, the inflation is brought down to 2.32% after 12 quarters, but at a cost terminal rate of 11.7% and the output loss of 8.48% that is much higher than for all other scenarios (Figures A3 and A4 show the path of inflation and interest rate under different cases).

So, what are the main conclusions from this exercise so far? First, quite obviously, it is always welfare improving if, despite all uncertainties with regard to the way the economy operates and economic agents behave, the central banks still get it right. But second, if they get it wrong from the beginning, central banks can still achieve outcomes that are close to optimal if they, seeing that their initial assumptions do not hold, change them and also modify their policy stance according to the new assumptions, and the sooner they adjust their policy to the new realities, the smaller the potential welfare loss. Conversely, sticking with the initial policy stance also in the face of changing evidence regarding the state of the economy implies the need for larger policy adjustments when they cannot longer be avoided, but these larger policy adjustments come with risks of more significant welfare losses.

Table 3: Terminal rate, inflation, and output loss under switching scenarios. Notes: rows represent scenarios assuming different values of the inflation coefficient in the Taylor rule, ψ_p =1.5 (dovish reaction); and ψ_p =4 (hawkish reaction). HE features backward-looking expectations and elements of learning. Correct/false indicates if the model used by the central bank describes the economy correctly.

Scenarios	**Terminal monetary policy rate**	**Inflation after 12 quarters**	**Inflation after 16 quarters**	**Cumulative output loss**
Case 1. (ψ_p=1.5, RE false)	3.41	4.77	4.16	1.21
Case 2. (ψ_p=4, HE correct)	7.19	2.18	2.11	5.00
Case 3. (ψ_p=1.5, RE false and after Q4 ψ_p=4, HE correct)	7.98	2.26	2.07	6.32
Case 4. (ψ_p=1.5, RE false and after Q8 ψ_p=10, HE correct)	11.7	2.32	1.77	8.48

4.6 Higher-for-longer or further hike?

In the previous section, we mostly discussed the potential risks of central bank policy reactions that is too late, too much. But the analytical framework also allows for analysing a different approach to monetary policy reaction: namely a "higher for longer" option. Facing high inflation, central banks have two possibilities: (1) to raise the interest rate until inflation comes down to an acceptable level; or (2) to commit to keep the interest rate at a lower, but still restrictive level for a longer period. The second option may be preferable from the financial stability perspective as high interest rates may create problems for the financial system by reducing the value of fixed income assets. The failure of some regional US banks at the beginning of 2023 was an example of such troubles.

We simulate scenarios where the central bank keeps the peak interest rate, R_{max}, at 3.5% and 4% until the period when the interest rate implied by the conventional Taylor rule is lower than these levels. We consider cases of ψ_p=2 and ψ_p=4 in the RE model.[2], [3] Figure 7 shows the paths of the interest rate under the benchmark scenario and scenarios for which the interest rate does not exceed 3.5% and 4%.

[2]Under such specification of the policy reaction function, the model becomes nonlinear and for the benchmark, ψ_p =1.5, as well for the HE models, the numerical solution cannot be found.

[3]For technical details of the scenarios implementation, see Appendix E

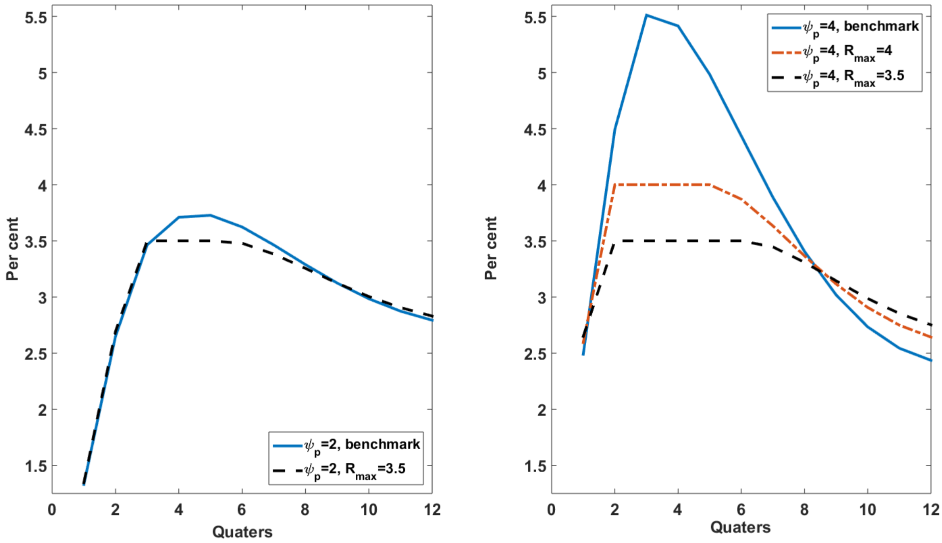

Figure 7: Interest rate path for higher-for-longer and linear Taylor rule scenarios. Notes: ψ_p =2 (dovish reaction); and ψ_p =4 (hawkish reaction). A correctly chosen RE model is assumed. R_{max}, represents a ceiling for the monetary policy rate (set at 3.5% and 4%) which is kept until the interest rate implied by the conventional Taylor rule is lower than these levels.

Table 4 shows that changes in cumulative inflation[4] for 12 quarters (log-differences in prices for a three years period) are not considerable across neither of the scenarios. In terms of output loss the higher-for-longer policies seem to perform better compared with linear Taylor rule implied policies. To sum up, in a model economy the higher-for-longer policy may provide some advantages in terms of cumulative output loss with a moderate increase in cumulative inflation.

Table 4: Interest rate path for higher-for-longer and linear Taylor rule scenarios. Notes: ψ_p =2 (dovish reaction); and ψ_p =4 (hawkish reaction). A correctly chosen RE model is assumed. R_{max}, represents a ceiling for the monetary policy rate (set at 3.5% and 4%) which is kept until the interest rate implied by the conventional Taylor rule is lower than these levels.

Scenarios	**Terminal monetary policy rate**	**Duration HFL period**	**Cumulative inflation**	**Cumulative output loss**
Dovish reaction (ψ_p =2)				
TR	3.73	-	10.90	3.23
HFL, R_{max} =3.5	3.50	3	11.05	3.03
Hawkish reaction (ψ_p =4)				
TR	5.51	-	9.68	4.84
HFL R_max=4	4.00	4	10.19	3.97
HFL R_max=3.5	3.50	5	10.30	3.49

[4]Since the difference between cumulative inflations of different policies is already quite small, we do not consider the difference in inflation levels.

5 Conclusions

This paper analyses how different sources of uncertainties central banks face shape the macroeconomic outcomes in the Smets-Wouters type Smets and Wouters (2003) and Smets and Wouters (2007) DSGE model. It **first** compares the optimal policy response given different forms of expectations formation, namely backward-looking expectations with elements of learning, and RE. **Second**, it considers that the central bank may use wrong expectations formation models to define a future interest rate path and then see what this means for inflation and output. **Third**, it analyzes different forms of potential nonlinearities in the conduct of monetary policy.

Overall, the analysis suggests that an aggressive monetary policy is preferable and more successful in curbing the high inflation rate for both rational and hybrid expectations formation mechanisms. In the presence of uncertainty regarding expectations formation mechanism, the hawkish response to high inflation provides less expected loss for small weights assigned to output loss.

Overall, hawkish monetary policy is more robust to uncertainty regarding the expectations formation and the correct or wrong model is used by the central bank with the main mandate of price stability, i.e. less weighted cumulative output loss in the loss function. The underestimation of inflation persistence and the delay in aggressive response to inflation result in much higher terminal rate and cumulative loss in output. The results are obtained under the assumption of hybrid expectations only for one endogenous variable, namely for inflation. The application of hybrid expectations also to wages does not change the results considerably.

A Results of Bayesian estimation

Table 5: Results from Metropolis-Hastings (parameters). HPD inf is posterior density 5% interval, HPD sup is posterior density 95% interval

	Prior			Posterior			
	Dist.	Mean	Stdev.	Mean	Stdev.	HPD inf	HPD sup
persistence productivity shock, ρ_a	beta	0.500	0.2000	0.903	0.0283	0.8745	0.9312
persistence risk premium shock, ρ_b	beta	0.500	0.2000	0.327	0.0346	0.2921	0.3613
persistence spending shock, ρ_g	beta	0.500	0.2000	0.966	0.0043	0.9621	0.9708
persistence risk premium shock, ρ_i	beta	0.500	0.2000	0.923	0.0055	0.9174	0.9284
persistence monetary policy shock, ρ_r	beta	0.500	0.2000	0.500	0.0101	0.4899	0.5101
persistence price markup shock, ρ_p	beta	0.500	0.2000	0.873	0.0372	0.8360	0.9104
persistence wage markup shock, ρ_w	beta	0.500	0.2000	0.830	0.0681	0.7618	0.8979
coefficient on MA term price markup, μ_p	beta	0.500	0.2000	0.607	0.0855	0.5213	0.6923
coefficient on MA term wage markup, μ_w	beta	0.500	0.2000	0.351	0.0896	0.2617	0.4409
investment adjustment cost, φ	norm	4.000	1.5000	5.108	0.9148	4.1935	6.0232
risk aversion, σ_c	norm	1.500	0.3750	1.312	0.0556	1.2567	1.3679
external habit degree, λ	beta	0.700	0.1000	0.848	0.0051	0.8424	0.8526
Calvo parameter wages, ξ_w	beta	0.500	0.1000	0.673	0.0030	0.6701	0.6760
Calvo parameter prices, ξ_p	beta	0.500	0.1000	0.684	0.0138	0.6705	0.6980
Frisch elasticity, σ_l	norm	2.000	0.7500	0.784	0.3849	0.3988	1.1685
Indexation to past wages, ι_w	beta	0.500	0.1500	0.262	0.0098	0.2524	0.2720
Indexation to past prices,ι_p	beta	0.500	0.1500	0.167	0.0303	0.1368	0.1975
capacity utilization cost,ψ	beta	0.500	0.1500	0.579	0.1726	0.4063	0.7516
fixed cost share,ϕ_p	norm	1.250	0.1250	1.323	0.1135	1.2091	1.4361
steady state hours,$\bar{l}$	norm	0.000	2.0000	0.436	0.0347	0.4016	0.4710
technology on exogenous spending, ρ_{ga}	norm	0.500	0.2500	0.312	0.0401	0.2721	0.3522
capital share,α	norm	0.300	0.0500	0.290	0.0120	0.2782	0.3021
parameter in employment equation,ξ_e	beta	0.500	0.2800	0.807	0.0090	0.7978	0.8158

Table 6: Results from Metropolis-Hastings (standard deviation of structural shocks)

	Prior			Posterior			
	Dist.	Mean	Stdev.	Mean	Stdev.	HPD inf	HPD sup
Productivity shock, σ^a	invg	0.100	3.0000	0.624	0.0434	0.5810	0.6677
Risk premium shock, σ^b	invg	0.100	3.0000	0.129	0.0184	0.1104	0.1472
Spending shock, σ^g	invg	0.100	3.0000	0.265	0.0030	0.2620	0.2680
Investment-specific technology shock, σ^i	invg	0.100	3.0000	0.222	0.0355	0.1868	0.2579
Monetary policy shock, σ^m	invg	0.100	3.0000	0.336	0.0062	0.3295	0.3419
Price markup shock, σ^p	invg	0.100	3.0000	0.087	0.0018	0.0849	0.0884
Wage markup shock, σ^w	invg	0.100	3.0000	0.073	0.0173	0.0554	0.0900

B Simulation results of HE for $\alpha = \delta = 0.5$

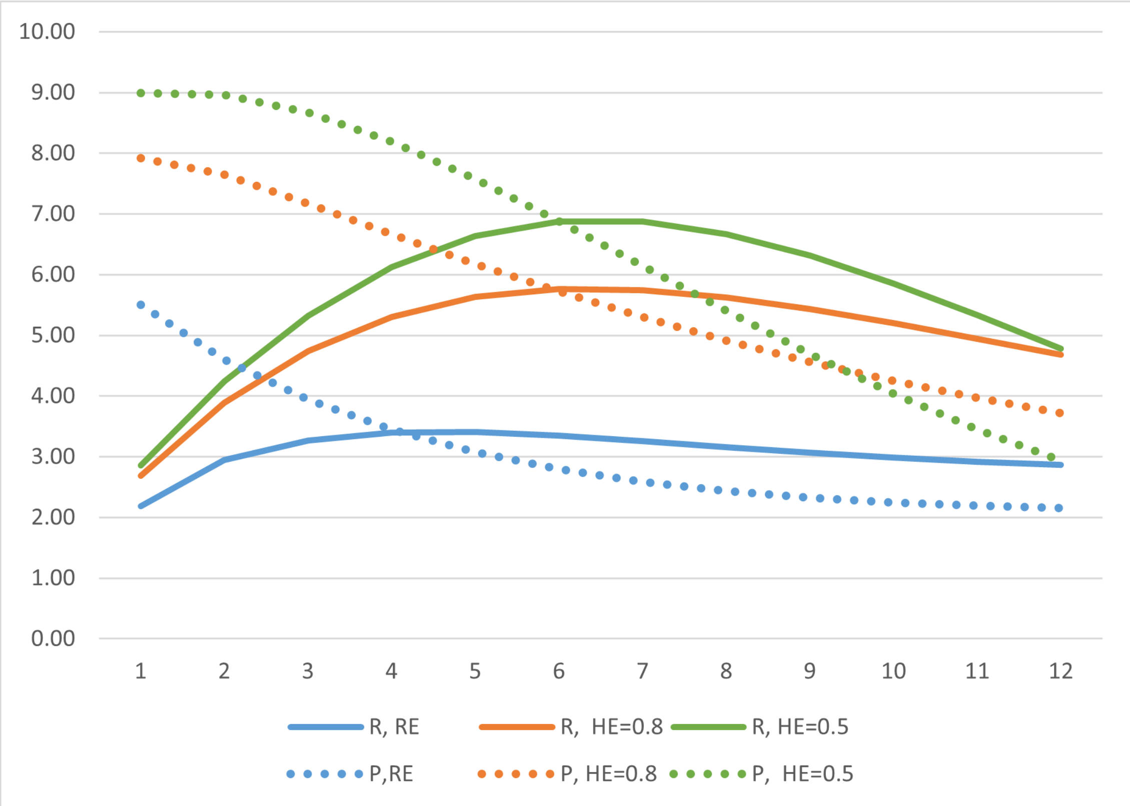

Figure 8: Interest rate and inflation under different degree of backward-looking behavior

Figure 8 shows the path of the interest rate and inflation for various degrees of backward-looking behaviour – RE ($\alpha = \delta = 0$), HE ($\alpha = \delta = 0.8$), and HE ($\alpha = \delta = 0.5$). Overall, for the moderate backward-looking behaviour ($\alpha = \delta = 0.5$), the path of interest rate and inflation lies between RE and HE ($\alpha = \delta = 0.8$). However, the inflation rate for HE ($\alpha = \delta = 0.8$) is lower than for HE ($\alpha = \delta = 0.5$) due to the higher interest rate.

C Expected loss functions for the RE and HE worlds and probabilities p=0.5 for each model used by the CB.

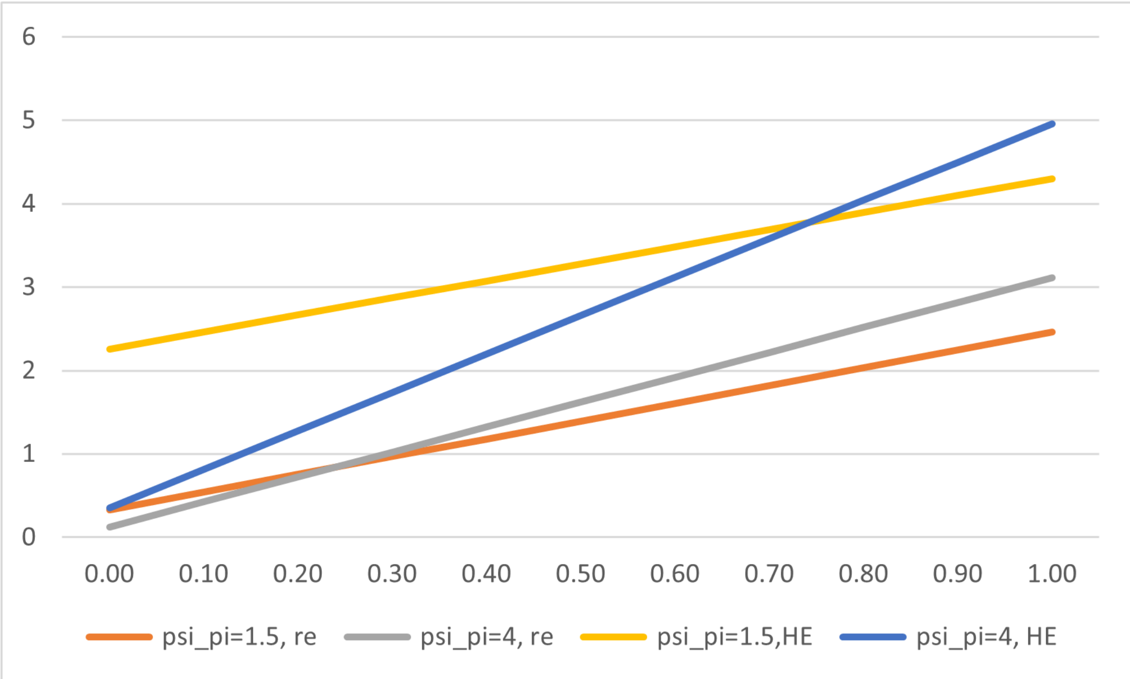

Figure 9: The dependence of the expected loss functions on of the weight w_y attached to the cumulative output loss for the hawkish (red line) and dovish (blue line) central banks.

Figure 9shows the expected loss functions with equal probability of the model used by the CB (RE or HE). In the RE world, the hawkish policy (gray line) provides better outcome than the dovish one (orange line) if the weight for output loss is less than 0.3. In the HE world, the hawkish policy (blue line) provides better outcome than the dovish one (orange line) if the weight for output loss is less than 0.8. For small weights for output loss, the expected loss is much lower for the aggressive reaction function than for the moderate one.

D

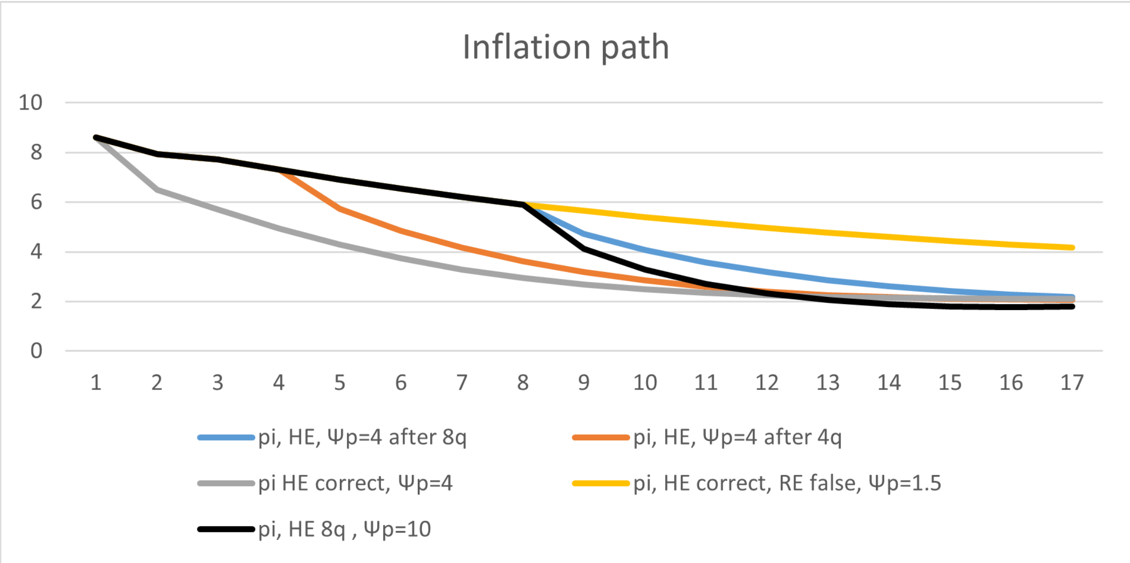

Figure 10: **The inflation rate path for a dovish central bank that had used the wrong RE model for 4 or 8 quarters, then switched to the right HE model and become hawkish**

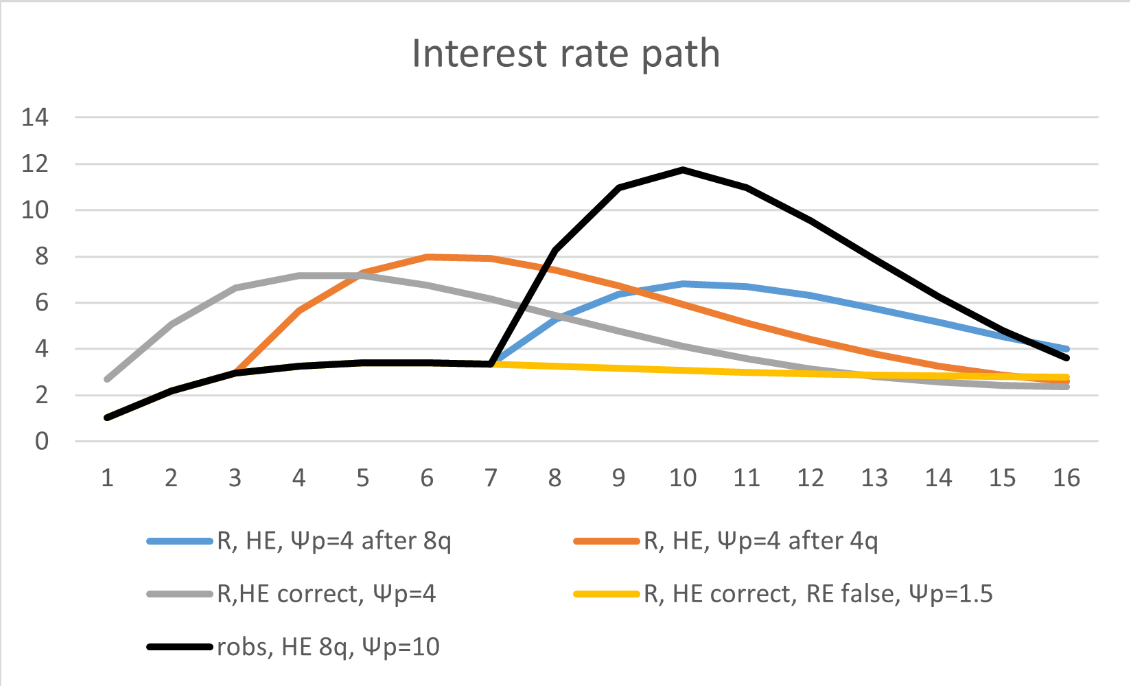

Figure 11: **The interest rate path for a dovish central bank that had used the wrong RE model for 4 or 8 quarters, then switched to the right HE model and become hawkish**

E Methotology and algorithms of implementation for scenarios simulation

The initial conditions for dynamics of macrovariables are obtained by employing the Kalman filter provided by Dynara's calibrated smoother. Estimated mean values are used as calibrated parameters. In further simulations, the initial conditions are defined in Dynara's block histval.

In Section 4.2, we examine scenarios in which the central bank uses an incorrect model to determine the future path of the nominal interest rate. To simulate this scenario, two Dynare files are run in order. For example, in a scenario where the correct model is HE, but the central bank uses the incorrect RE model, the first file simulates the incorrect RE model, forecasts the future interest rate path, and saves it. The second file then simulates the behavior of households and firms, allowing them to compute conditional forecasts based on the constrained interest rate path generated in the simulation of the first file.

In Section 4.5, to simulate switching the model and the central bank reaction function for case 3 (4) we first forecast endogenous variables for the RE model for four (eight) quarters. Saving the values of state variables at the fourth quarter, next we use them as initial values for running the HE model for 12 quarters by employing the command *histval* in another dynare-file. Case 5 is similar to Case 4 except for running the HE model at the second stage by using the Taylor rule with coefficient $\psi_p = 10$. This value of the coefficient is chosen to provide the inflation rate being close to 2% after 12 quarters after the begginning of simulation (i.e. the start of running the RE model), avoiding inflation undershooting.

In Section 4.6, to simulate the higher-for-longer policy we use the Dynare's perfect foresight solver with the following nonlinear specification of the Taylor rule:

$$R_t = \max\left(\bar{R}_i, \rho R_{t-1} + (1-\rho)\left(r^* + \pi^* + \psi_p\left(\pi_t^{(4)} - \pi^*\right) + \psi_y y_t^{gap}\right)\right) \tag{6}$$

where $\bar{R}_i = 3.5$ or 4 are possible terminal rates of the central bank. The initial conditions, defined by the command *histval*, are the same as for other simulations.

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